PART III

GROWTH THEORY: THE ECONOMY IN THE VERY LONG RUN

Chapter 7: Economic Growth I: Capital Accumulation and Population Growth*

MACROECONOMICS

Seventh Edition

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*Slides based on Ron Cronovich's slides, adjusted by Marcel Bluhm for lecture in Macroeconomics at the Wang Yanan Institute for Studies in Economics at Xiamen University.

The Lessons of Growth Theory...

…can make a positive difference in the lives of hundreds of millions of people.

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and our government's policies

Learning Objectives

This chapter introduces you to understanding economic growth in relation to:

- the accumulation of capital ⇦
- the golden rule level of capital
- population growth

7.1) The Accumulation of Capital

➔ The Solow Growth Model (SGM)

- Due to Robert Solow, won Nobel Prize for contributions to the study of economic growth
- A major paradigm:
  - Widely used in policy making
  - Benchmark against which most recent growth theories are compared
- Looks at the determinants of economic growth and the standard of living in the long run

7.1) The Accumulation of Capital

➔SGM and its Difference to the Model in Chap. 3

1. ***K*** is no longer fixed:
   investment causes it to grow,
   depreciation causes it to shrink

2. ***L*** is no longer fixed:
   population growth causes it to grow

3. The consumption function is simpler

4. No ***G*** or ***T***
   (only to simplify presentation;
   we can still do fiscal policy experiments)

5. Cosmetic differences

7.1) The Accumulation of Capital

➔ SGM: Production Function

In aggregate terms: $\boldsymbol{Y = F(K, L)}$

Define: $\boldsymbol{y = Y/L}$ = output per worker

$\boldsymbol{k = K/L}$ = capital per worker

Assume constant returns to scale:

$\boldsymbol{zY = F(zK, zL)}$ for any $\boldsymbol{z} > 0$

Pick $\boldsymbol{z = 1/L}$. Then

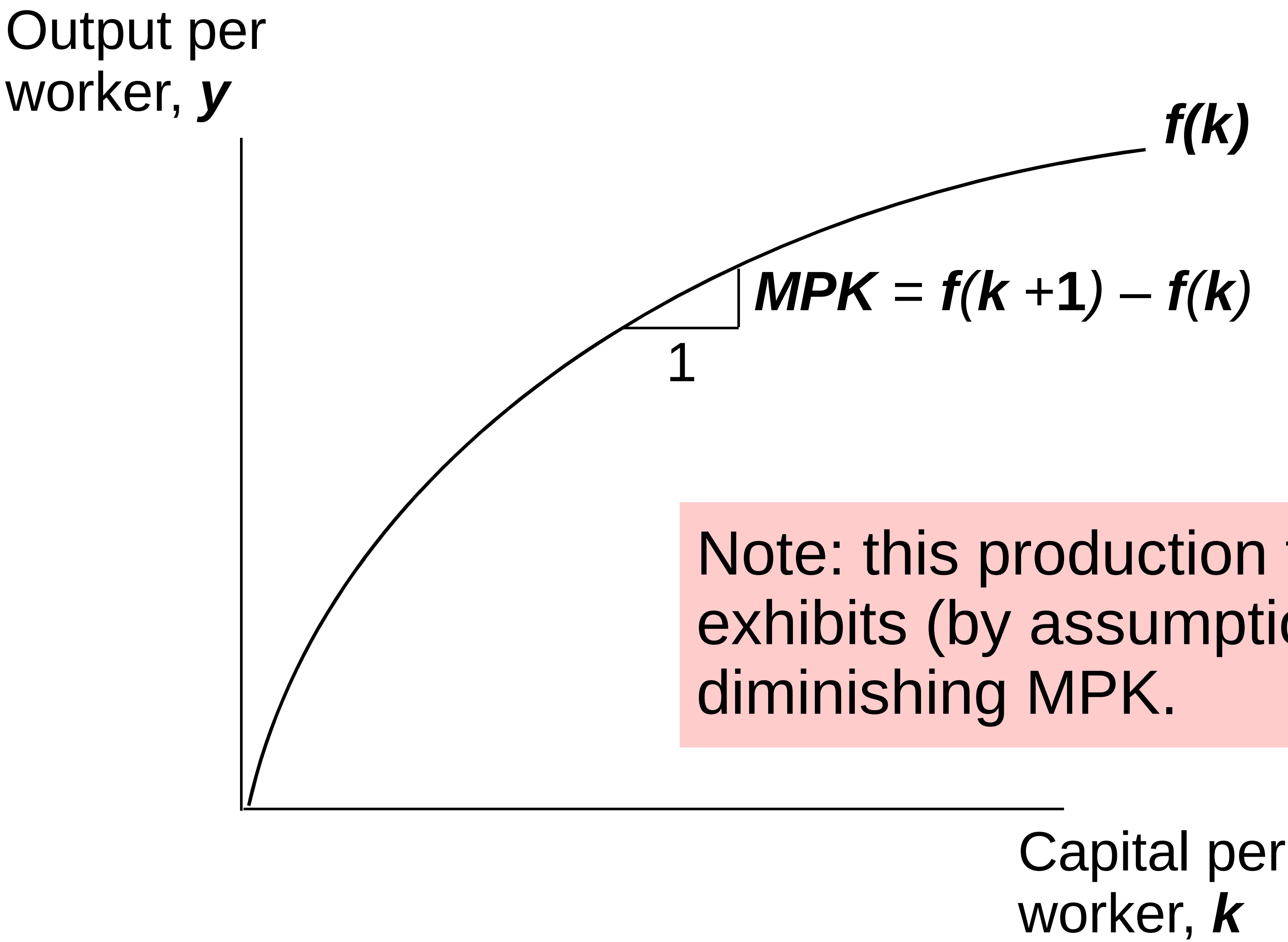
$$\boldsymbol{Y/L = F(K/L, 1)}$$

$$\boldsymbol{y = F(k, 1)}$$

$$\boldsymbol{y = f(k)} \quad \text{where } \boldsymbol{f(k) = F(k, 1)}$$

7.1) The Accumulation of Capital

➔ SGM: Production Function



Note: this production function exhibits (by assumption) a diminishing MPK.

7.1) The Accumulation of Capital

➔ SGM: National Income Identity

$\boldsymbol{Y} = \boldsymbol{C} + \boldsymbol{I}$ (remember, no $\boldsymbol{G}$)

In "per worker" terms: $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{i}$
where $\boldsymbol{c} = \boldsymbol{C}/\boldsymbol{L}$ and $\boldsymbol{i} = \boldsymbol{I}/\boldsymbol{L}$

7.1) The Accumulation of Capital

➔ SGM: Consumption Function

s = the saving rate, the fraction of income that is saved
(***s*** is an exogenous parameter)

Note: ***s*** is the only lowercase variable that is not equal to its uppercase version divided by ***L.***

Consumption function: $\boldsymbol{c} = (\mathbf{1}-\boldsymbol{s})\boldsymbol{y}$
(per worker)

7.1) The Accumulation of Capital

➔ SGM: Saving and Investment

- Saving (per worker) $= \boldsymbol{y} - \boldsymbol{c}$

  $= \boldsymbol{y} - (\mathbf{1}-\boldsymbol{s})\boldsymbol{y}$

  $= \boldsymbol{sy}$

- National income identity is $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{i}$

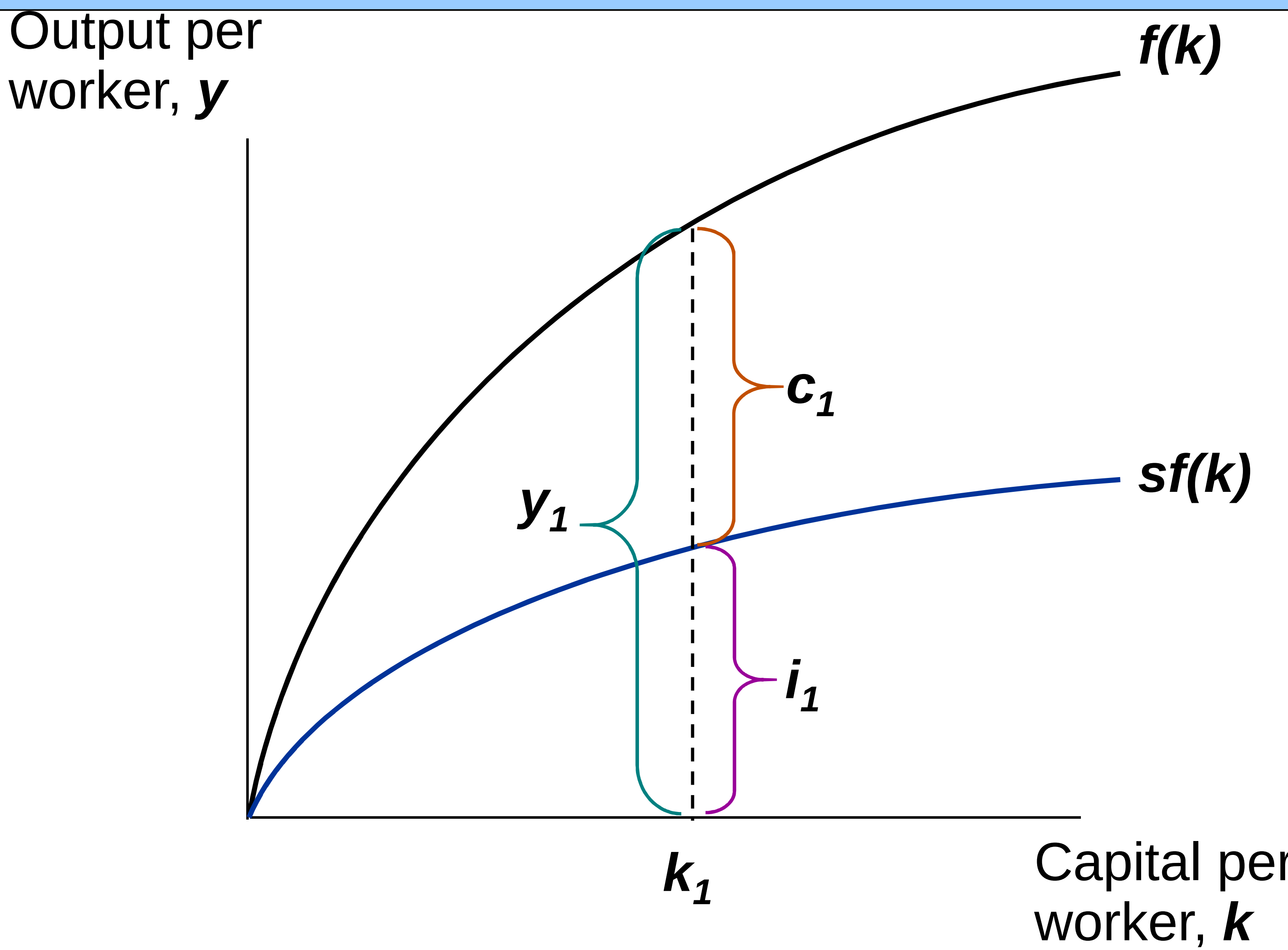
- Rearrange to get: $\boldsymbol{i} = \boldsymbol{y} - \boldsymbol{c} = \boldsymbol{sy}$

(investment = saving, like in chap. 3!)

- Using the results above yields $\boldsymbol{i} = \boldsymbol{sy} = \boldsymbol{sf(k)}$

7.1) The Accumulation of Capital

➔ SGM: output, Consumption, and Investment

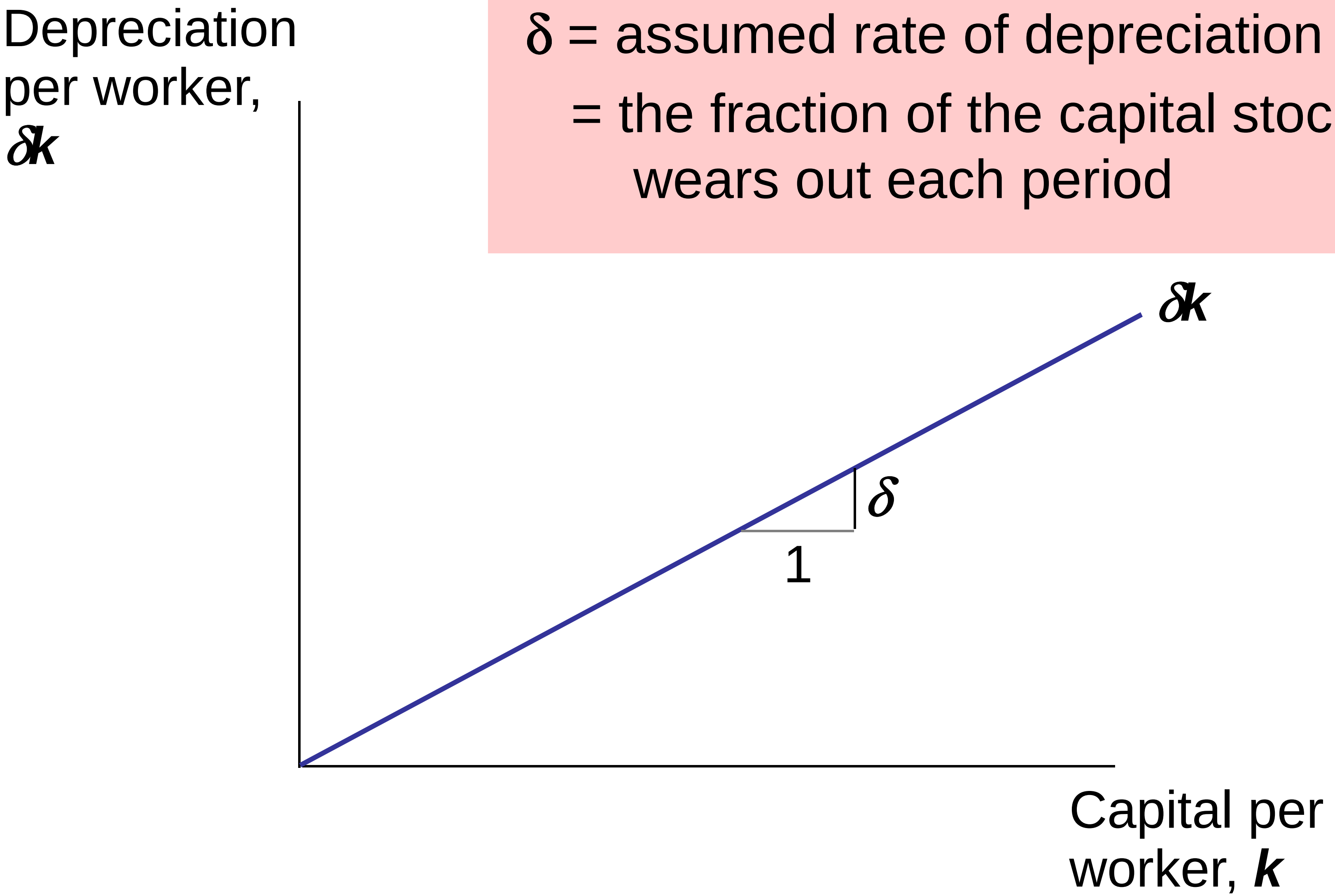


7.1) The Accumulation of Capital

➔ SGM: Depreciation of Capital

δ = assumed rate of depreciation

= the fraction of the capital stock that wears out each period



7.1) The Accumulation of Capital

➔ SGM: Capital Accumulation

- Basic idea: Investment increases the capital stock, depreciation reduces it.

- Change in capital stock = investment – depreciation

$$\Delta k \quad = \quad i \quad - \quad \delta k$$

- Since $i = sf(k)$, this becomes:

$$\Delta k = s\,f(k) - \delta k$$

7.1) The Accumulation of Capital

➔ SGM: Equation of Motion for k

$$\Delta \boldsymbol{k} = \boldsymbol{s\, f(k)} - \boldsymbol{\delta k}$$

- The Solow model's central equation
- Determines behavior of capital over time…
- …which, in turn, determines behavior of all of the other endogenous variables because they all depend on ***k***. For example
  - income per person: $\boldsymbol{y = f(k)}$
  - consumption per person: $\boldsymbol{c = (1 - s)\, f(k)}$

7.1) The Accumulation of Capital

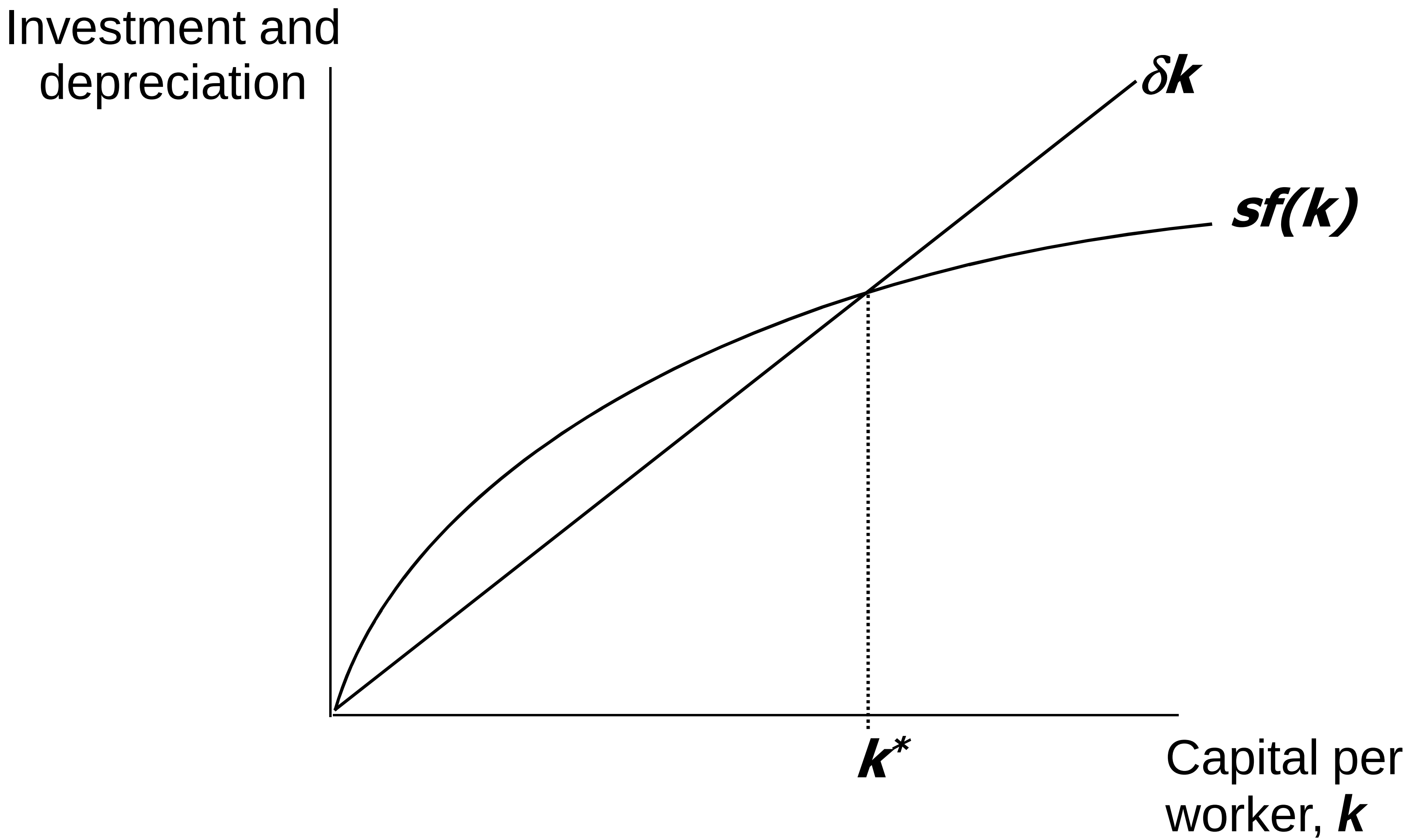
➔ SGM: The Steady State

$$\Delta \boldsymbol{k} = \boldsymbol{s}\, \boldsymbol{f(k)} - \delta \boldsymbol{k}$$

- If investment is just enough to cover depreciation, [$sf(k)=\delta k$]...
- ...then capital per worker will remain constant: $\Delta k = 0$.
- This occurs at one value of ***k***, denoted k^*, called the ***steady state capital stock***.

7.1) The Accumulation of Capital

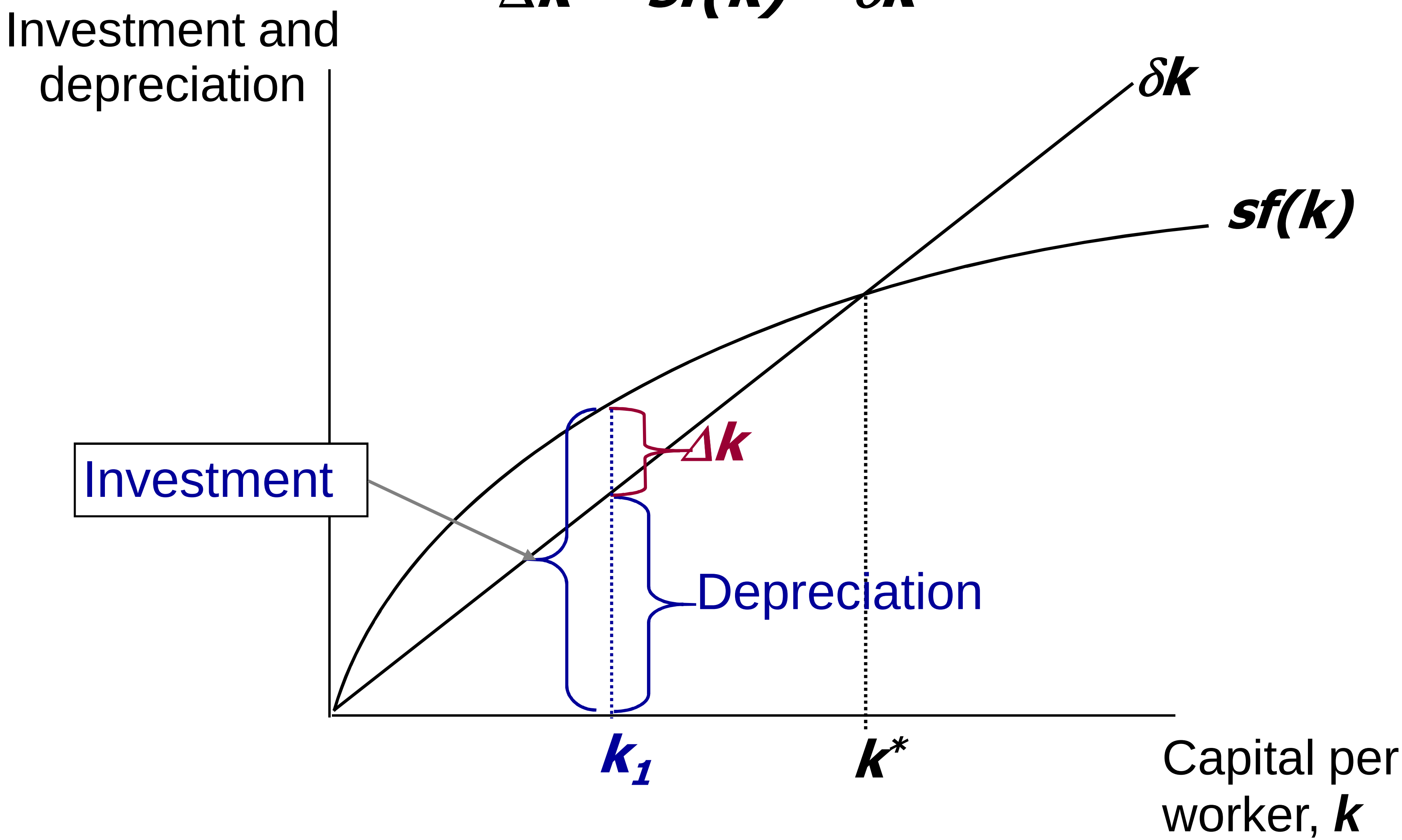
➔ SGM: The Steady State (ctd.)



7.1) The Accumulation of Capital

➔ SGM: Moving Toward the Steady State

$$\Delta k = sf(k) - \delta k$$



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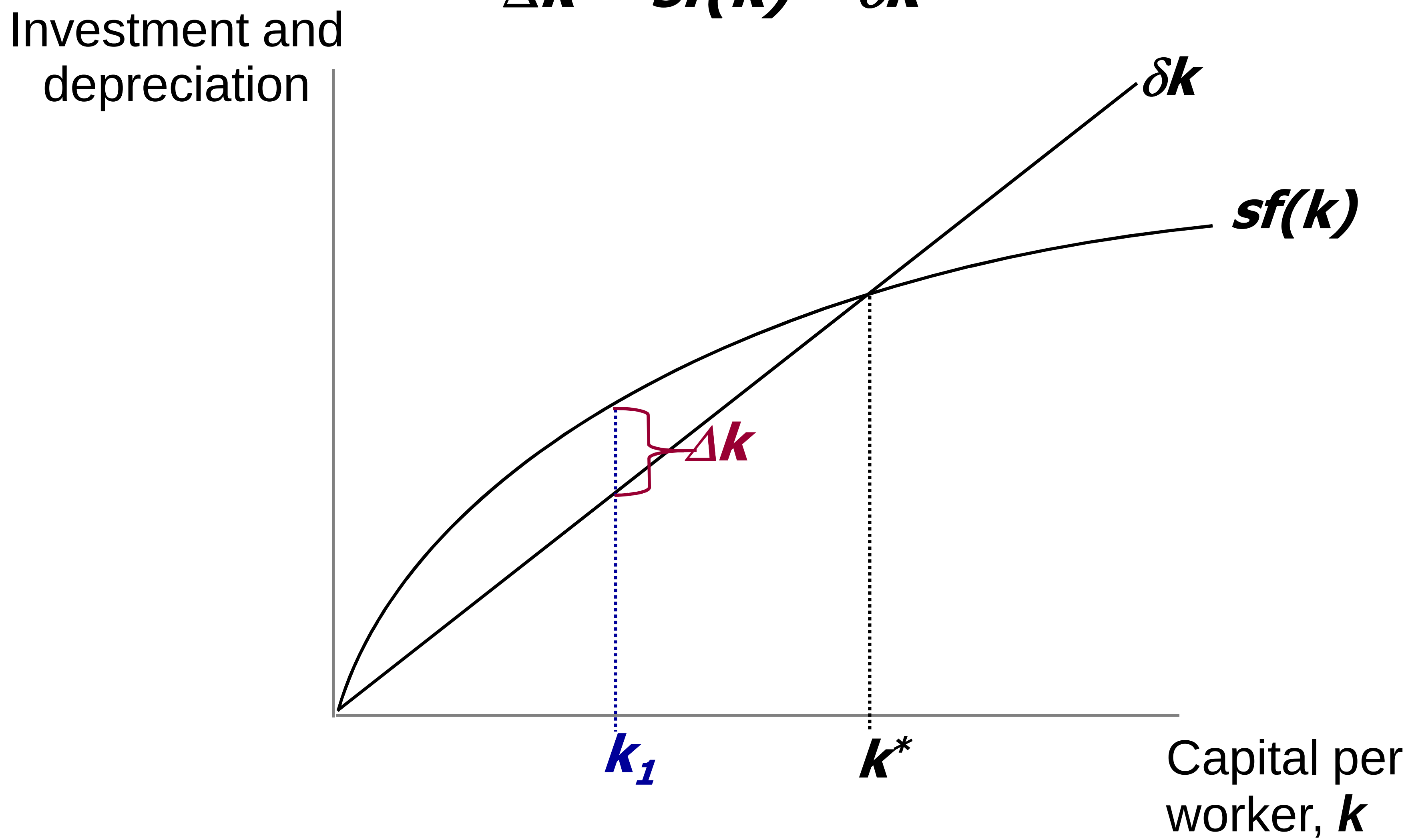

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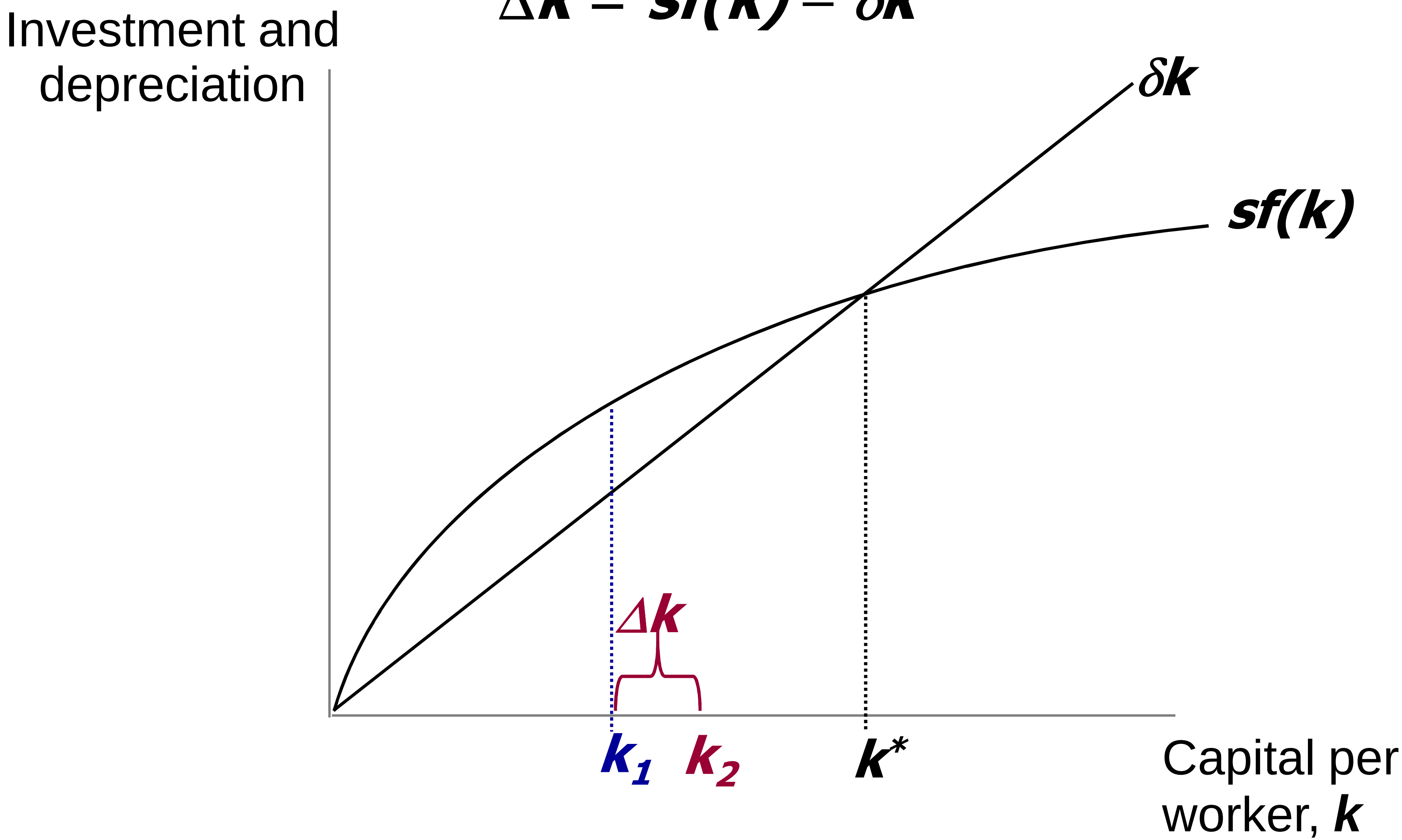

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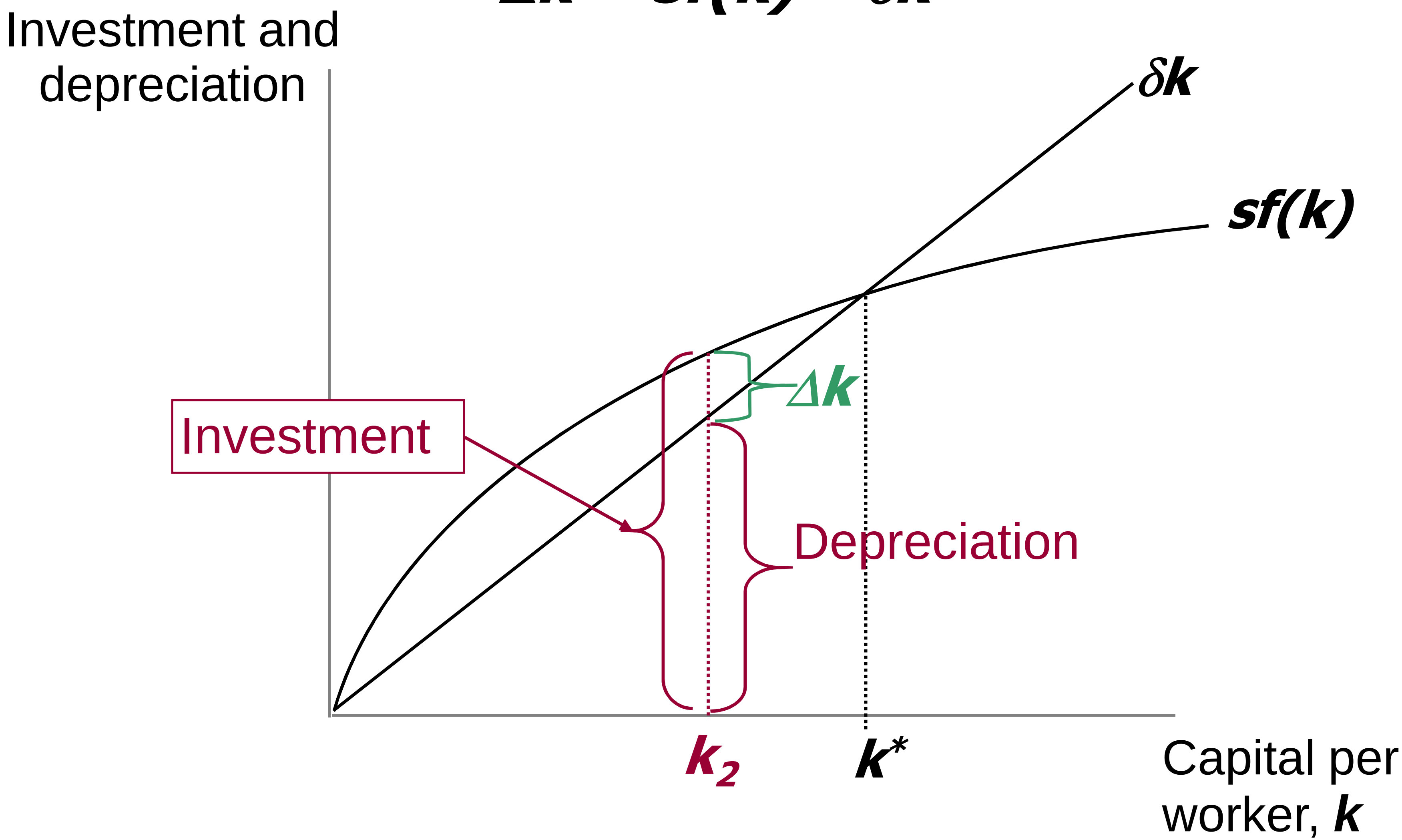

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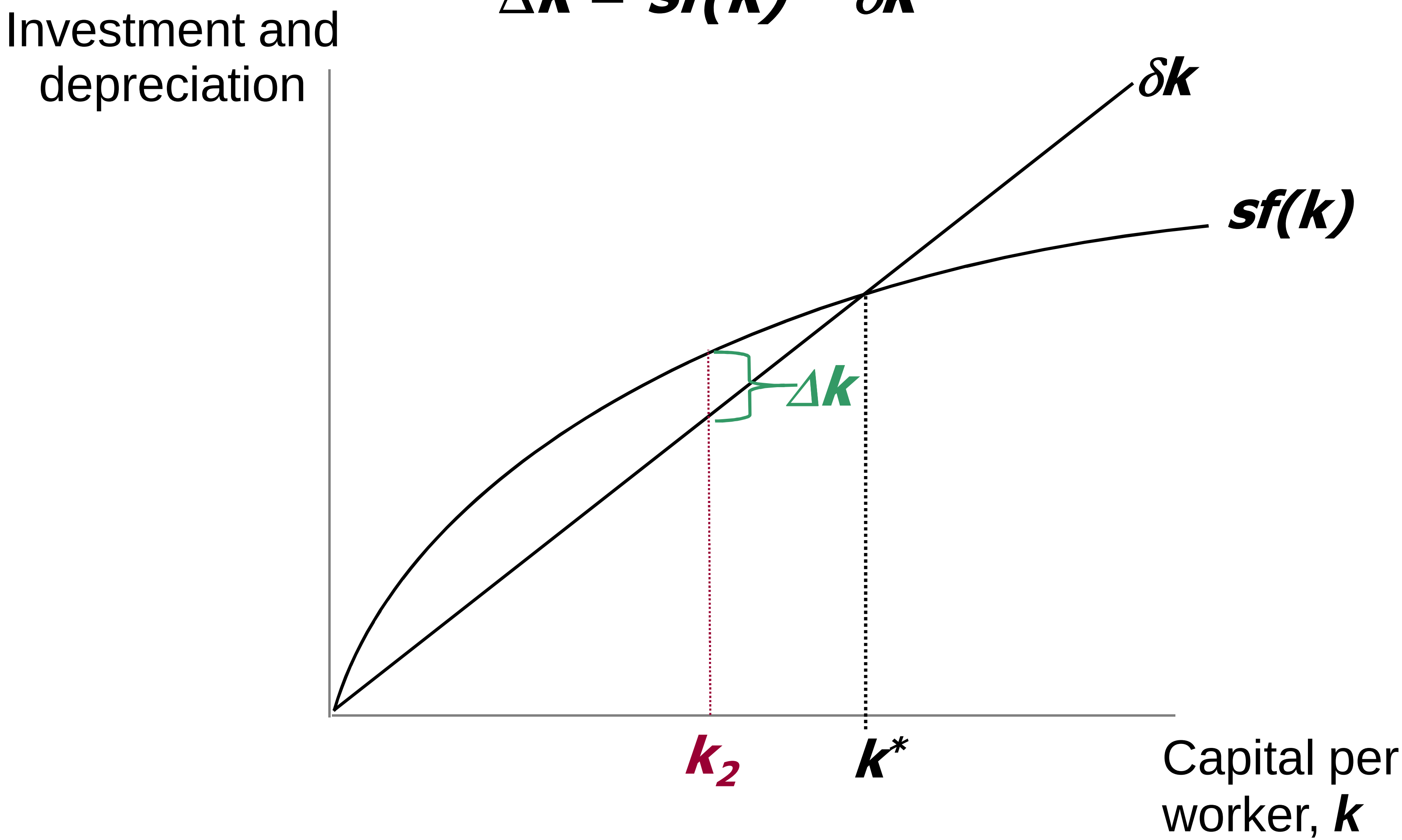

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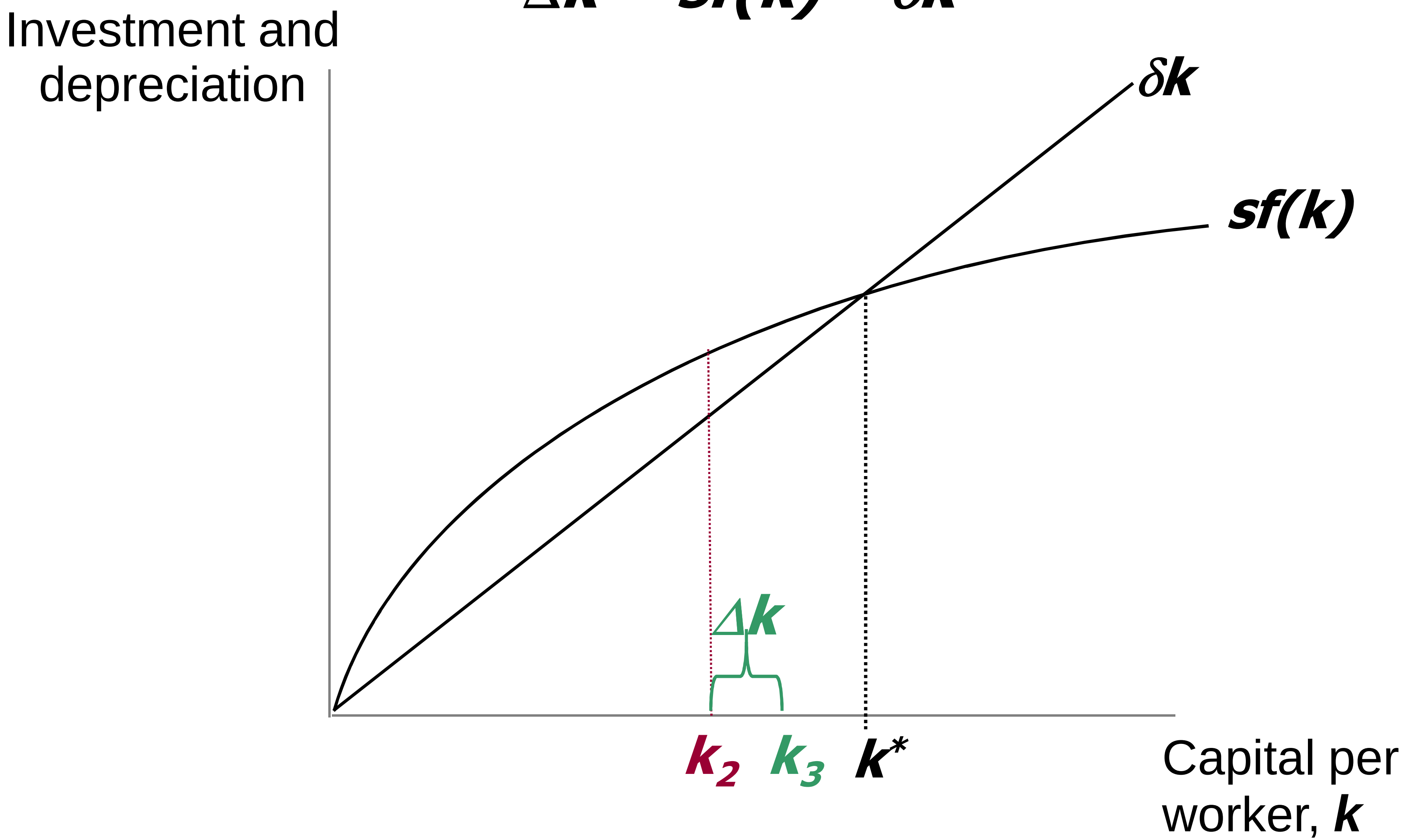

7.1) The Accumulation of Capital

➔ SGM: Moving Toward the Steady State (ctd.)

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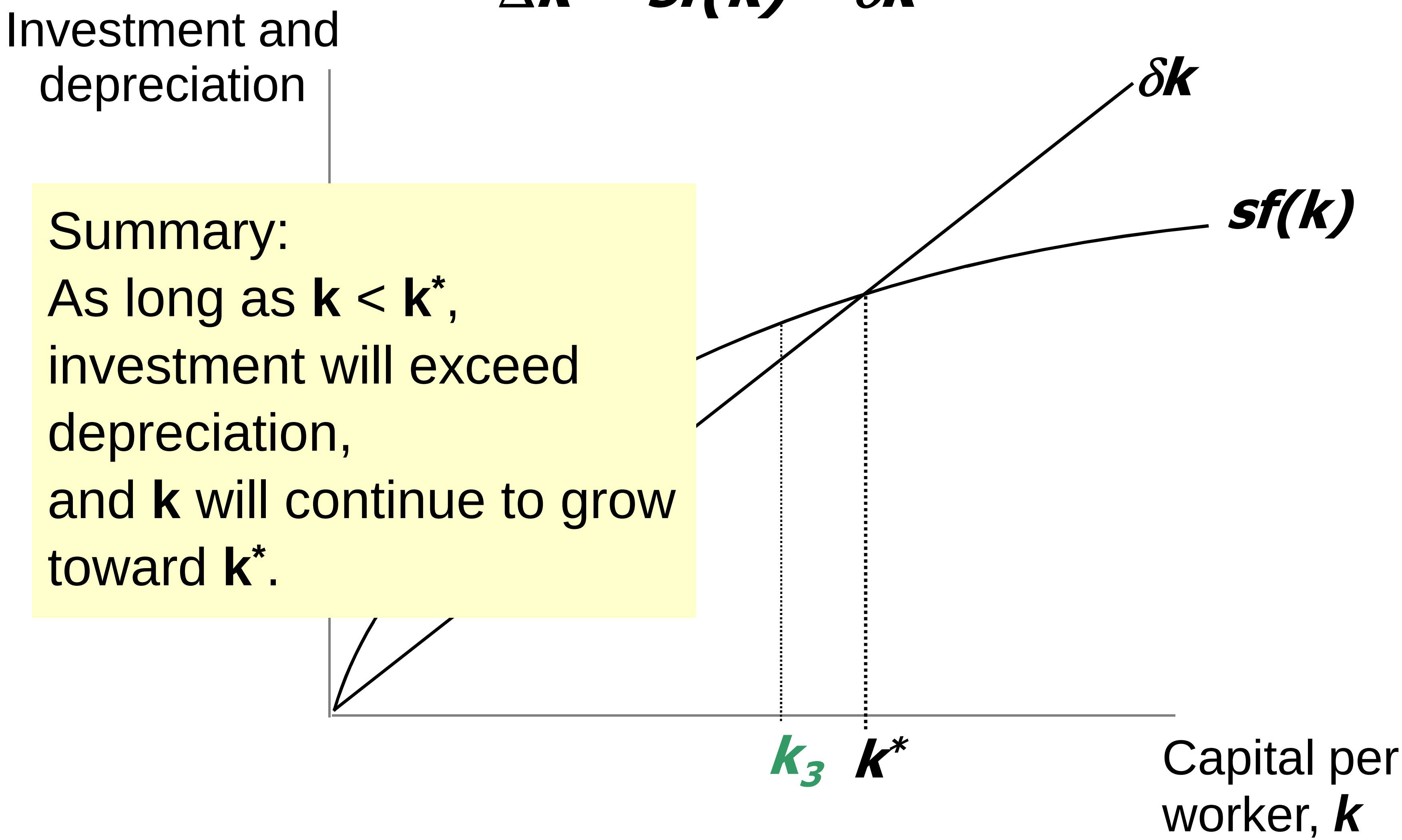


Summary:
As long as **k** < **k***,
investment will exceed depreciation,
and **k** will continue to grow toward **k***.

7.1) The Accumulation of Capital

➔SGM: 该你们了

- Draw the Solow model diagram, labeling the steady state ***k**** .

- On the horizontal axis, pick a value greater than ***k**** for the economy's initial capital stock. Label it $\boldsymbol{k_1}$.

- Show what happens to ***k*** over time. Does ***k*** move toward the steady state or away from it?

7.1) The Accumulation of Capital

➔ SGM: A Numerical Example

- Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2} L^{1/2}$$

- To derive the per-worker production function, divide through by ***L***:

$$\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}$$

- Then substitute ***y*** = ***Y/L*** and ***k*** = ***K/L*** to get

$$y = f(k) = k^{1/2}$$

7.1) The Accumulation of Capital

➔ SGM: A Numerical Example (ctd.)

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

7.1) The Accumulation of Capital

➔ SGM: Approaching the Steady State in Ex.

$$y = f(k) = k^{1/2}$$

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

Year	k	y	c	i	δk	Δk
1						
2						
3						
4						
…						
10						
…						
25						
…						
100						
…						
∞						

7.1) The Accumulation of Capital

➔ SGM: Solve for Steady State in Ex.

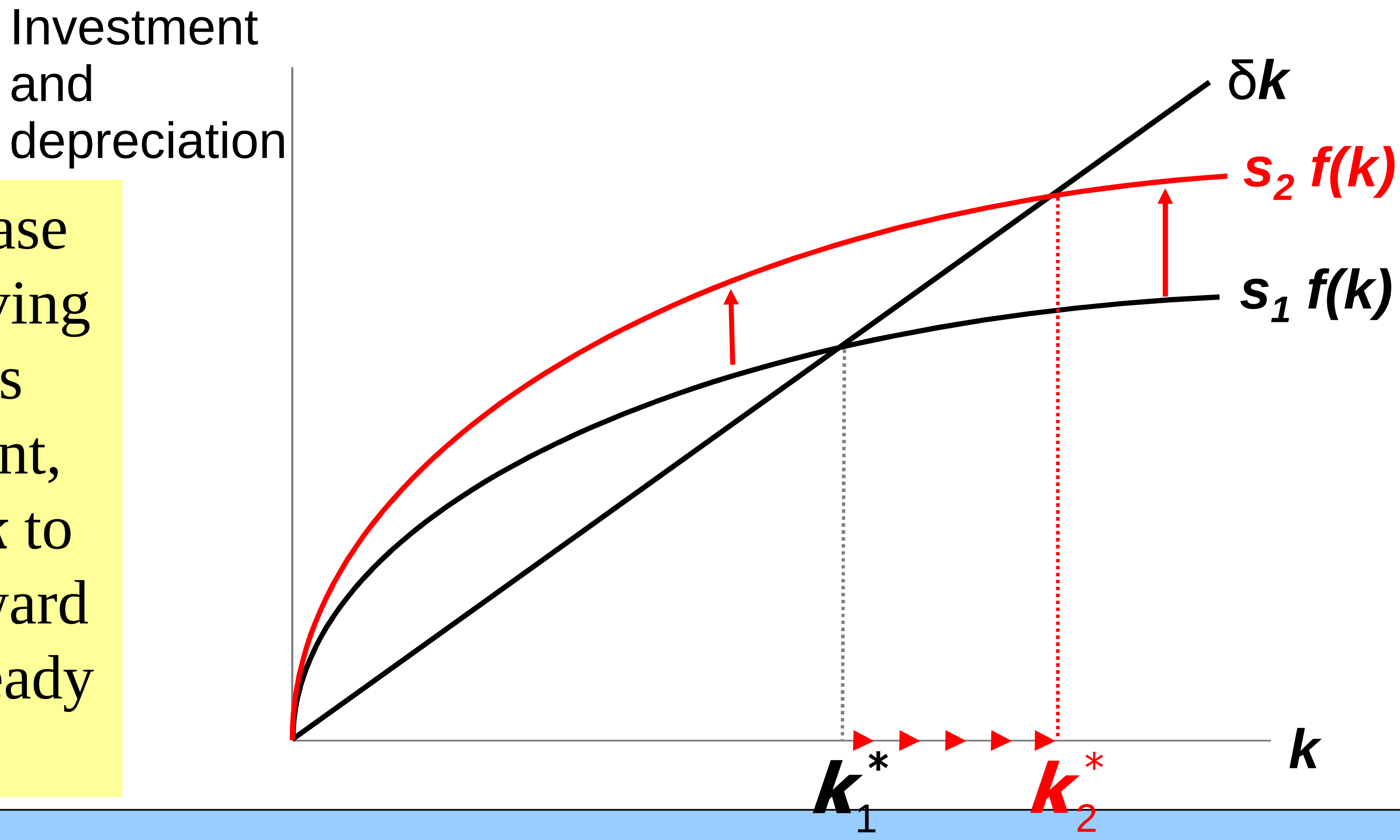
- Continue to assume $s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

- Use the equation of motion $\Delta k = s\,f(k) - \delta k$ to solve for the steady-state values of ***k***, ***y***, and ***c***.

7.1) The Accumulation of Capital

➔ SGM: Increase in the Savings Rate

An increase in the saving rate raises investment, causing ***k*** to grow toward a new steady state



7.1) The Accumulation of Capital

➔ SGM: Prediction of the Model w.r.t. $\Delta s>0$

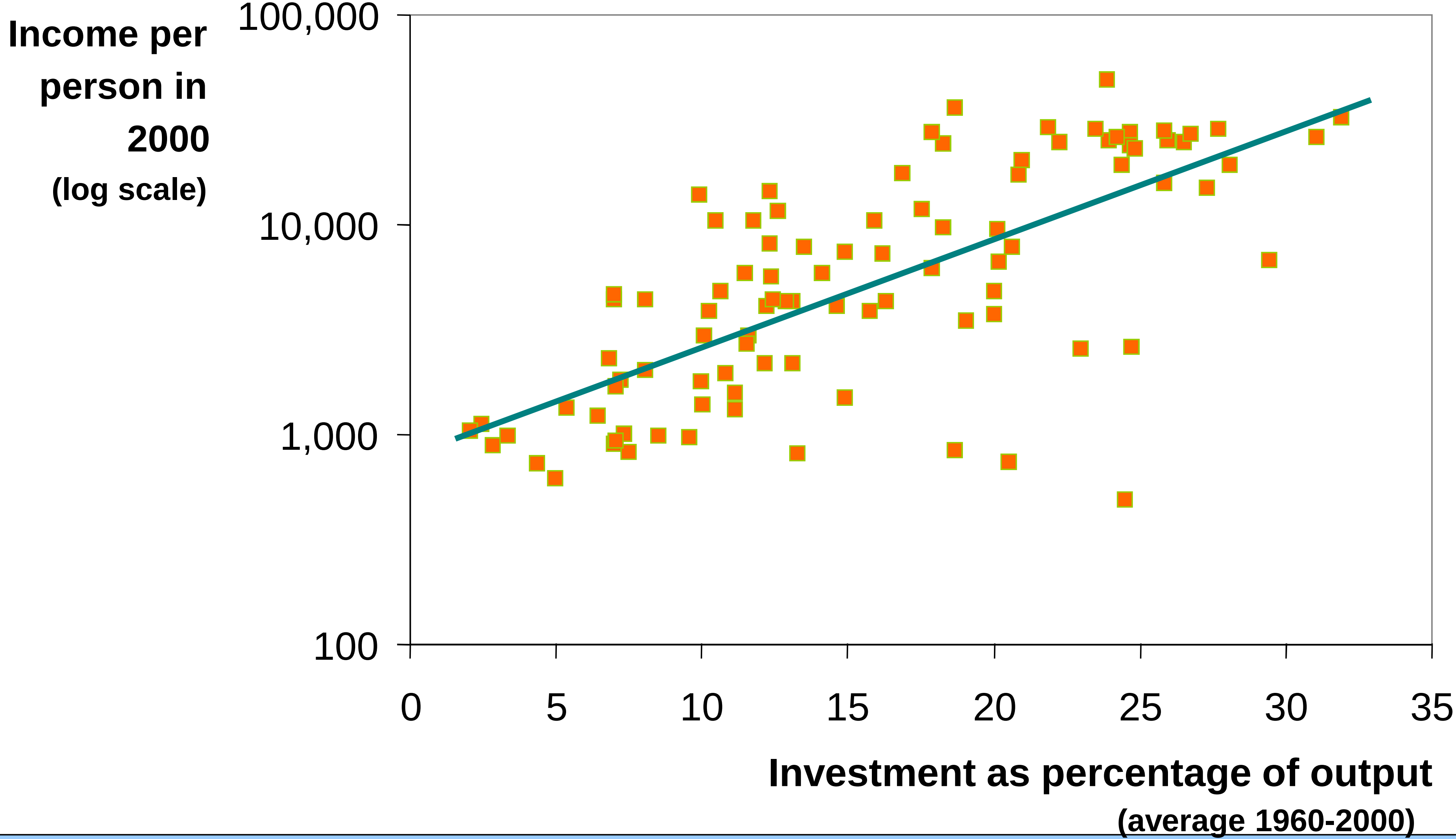
- Higher ***s*** $\Rightarrow$ higher $\boldsymbol{k^*}$.

- And since ***y*** = ***f(k)***, higher $\boldsymbol{k^*}$ $\Rightarrow$ higher $\boldsymbol{y^*}$.

- Thus, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.

7.1) The Accumulation of Capital

➔ SGM: International Evidence



Learning Objectives

This chapter introduces you to understanding economic growth in relation to:

- the accumulation of capital ✓
- the golden rule level of capital ⇦
- population growth

7.2) The Golden Rule of Capital

- Different values of ***s*** lead to different steady states. How do we know which is the "best" steady state?

- The "best" steady state has the highest possible consumption per person: ***c**** = (1–***s***) f(***k****).

- An increase in ***s*** leads to higher ***k**** and ***y****, which raises ***c*,*** reduces consumption's share of income (1–***s***), which lowers ***c*.***

- So, how do we find the ***s*** and ***k**** that maximize ***c****?

7.2) The Golden Rule of Capital

➔ SGM: Golden Rule Capital Stock

k^*_{gold} = the **Golden Rule level of capital**, the steady state value of k that maximizes consumption.

To find it, first express c^* in terms of k^*:

$$c^* = y^* - i^*$$

$$= f(k^*) - i^*$$

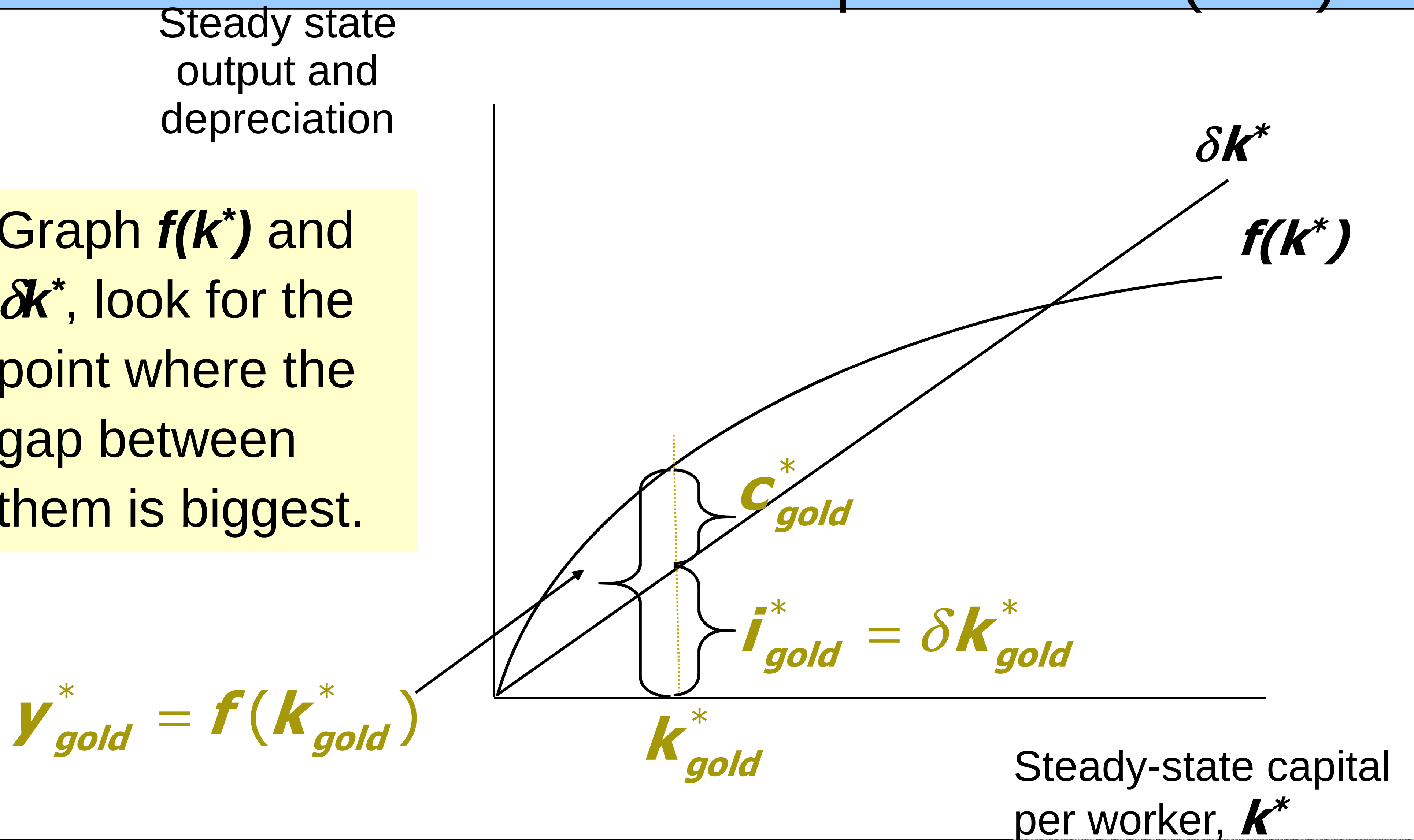
$$= f(k^*) - \delta k^*$$

In the steady state: $i^* = \delta k^*$ because $\Delta k = 0$.

7.2) The Golden Rule of Capital

➔ SGM: The Golden Rule Capital Stock (ctd.)

Graph ***f(k*)*** and δ***k****, look for the point where the gap between them is biggest.

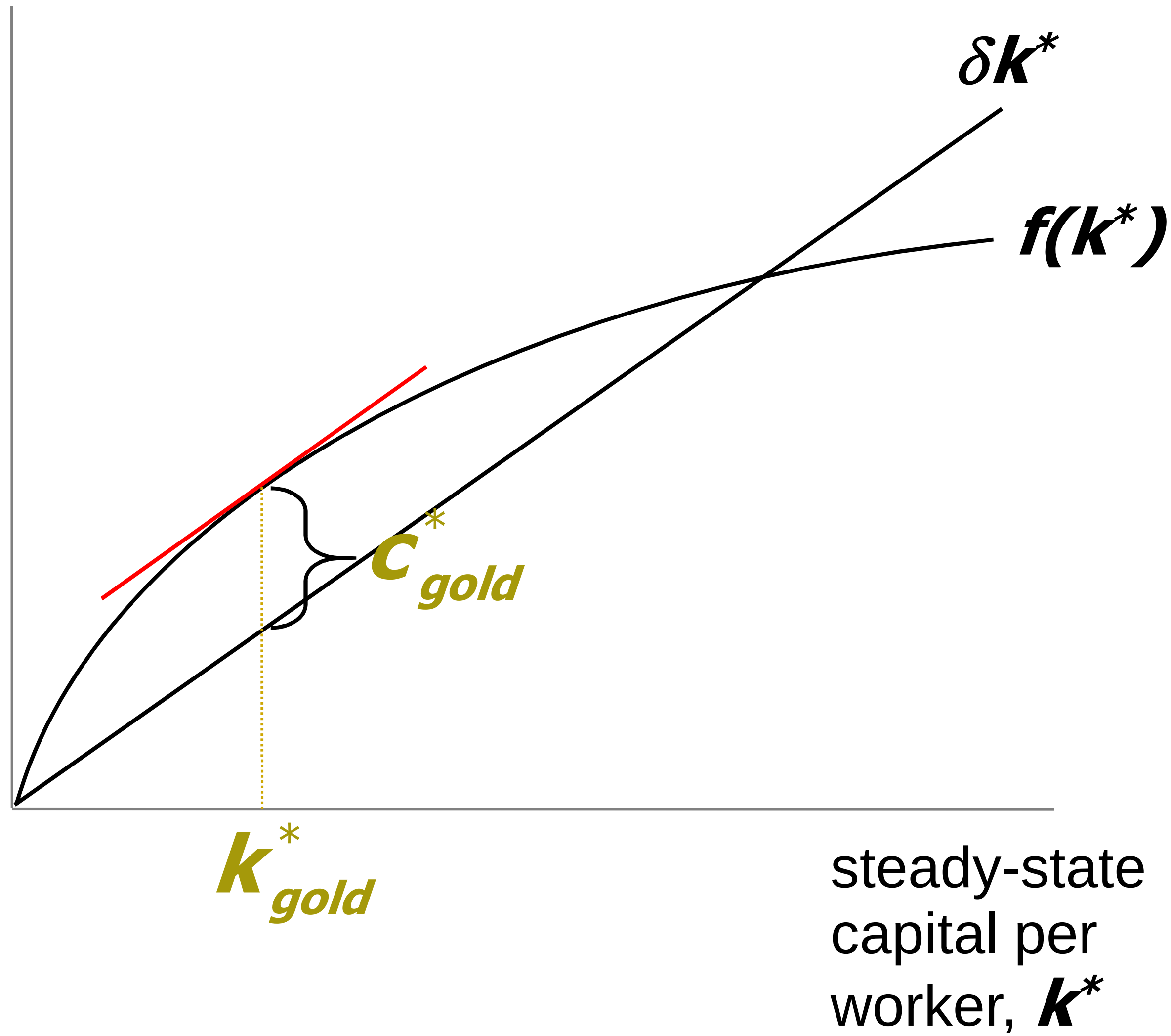


7.2) The Golden Rule of Capital

➔ SGM: The Golden Rule Capital Stock (ctd.)

$c^* = f(k^*) - \delta k^*$
is biggest where the slope of the production function equals the slope of the depreciation line:

MPK = δ



7.2) The Golden Rule of Capital

➔ SGM: Transition to Golden Rule Capital State

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

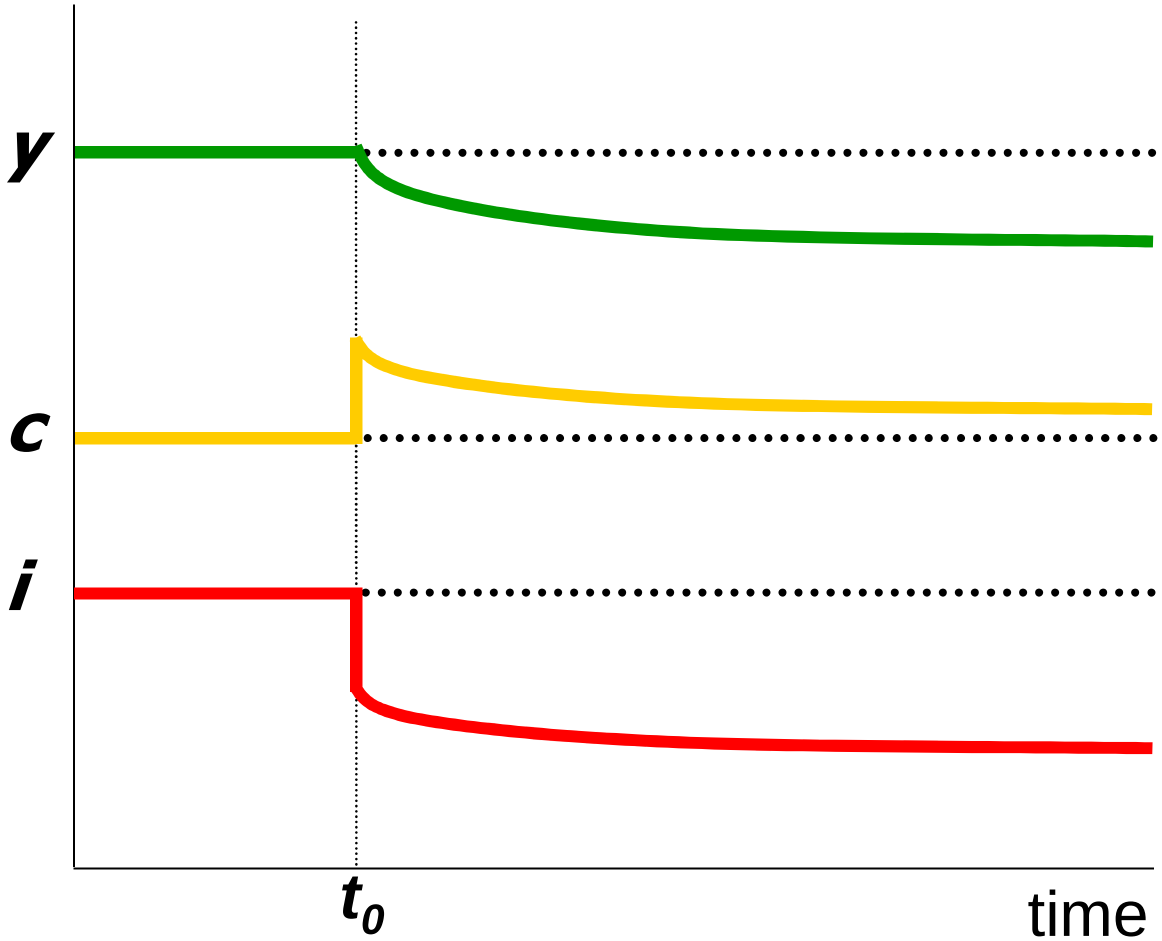
7.2) The Golden Rule of Capital

➔ SGM: Starting With too much Capital

If $\boldsymbol{k}^* > \boldsymbol{k}^*_{gold}$

then increasing $\boldsymbol{c}^*$ requires a fall in $\boldsymbol{s}$.

In the transition to the Golden Rule, consumption is higher at all points in time.

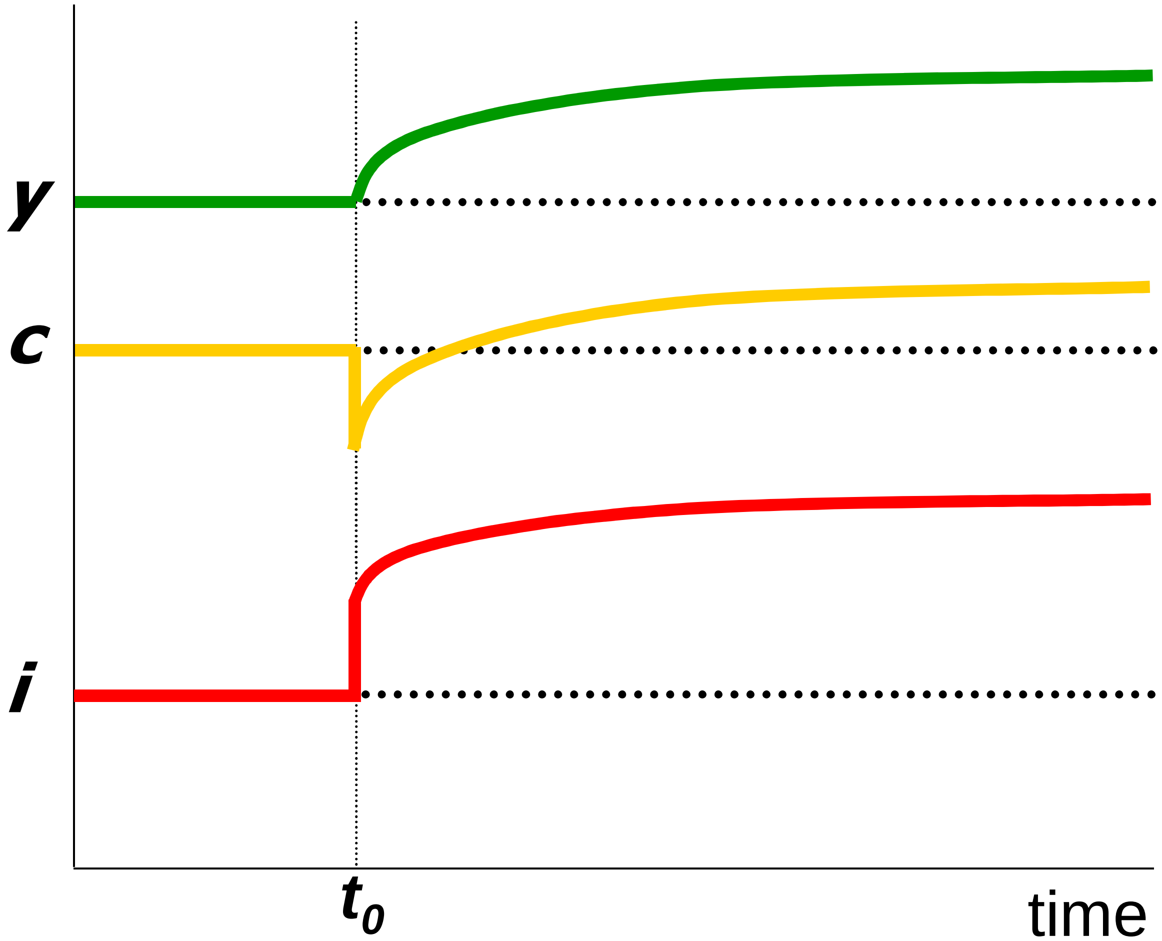


7.2) The Golden Rule of Capital

➔ SGM: Starting With too Little Capital

If $\boldsymbol{k}^* < \boldsymbol{k}^*_{gold}$
then increasing $\boldsymbol{c}^*$
requires an
increase in $\boldsymbol{s}$.

Future generations
enjoy higher
consumption,
but the current
one experiences
an initial drop
in consumption.



➔SGM: 该你们了

Consider an economy described by the production function $Y=F(K,L)=K^{1/3}L^{2/3}$:

a) Does this production function have constant returns to scale?

b) What is the per worker production function?

c) Assuming no population growth or technological progress, find the steady-state capital stock per worker, output per worker, and consumption per worker as a function of the saving rate and the depreciation rate.

Learning Objectives

This chapter introduces you to understanding economic growth in relation to:

- the accumulation of capital ✓
- the golden rule level of capital ✓
- population growth ⇐

7.3) Population Growth

➔ SGM and the Effect of Population Growth

- Assume that the population (and labor force) grow at rate ***n***. (***n*** is exogenous.)

$$\frac{\Delta L}{L} = n$$

- Example: Suppose ***L*** = 1,000 in year 1 and the population is growing at 2% per year (***n*** = 0.02).

- Then $\Delta L = n\,L = 0.02 \times 1{,}000 = 20$, so ***L*** = 1,020 in year 2.

7.3) Population Growth

→ SGM and Break-Even Investment

- $(\delta + n)k$ = **break-even investment**, the amount of investment necessary to keep ***k*** constant.

- Break-even investment includes:
  - δk to replace capital as it wears out
  - $n k$ to equip new workers with capital

- (Otherwise, ***k*** would fall as the existing capital stock would be spread more thinly over a larger population of workers.)

7.3) Population Growth

➔ SGM and the Equation of Motion for k

With population growth,
the equation of motion for ***k*** is

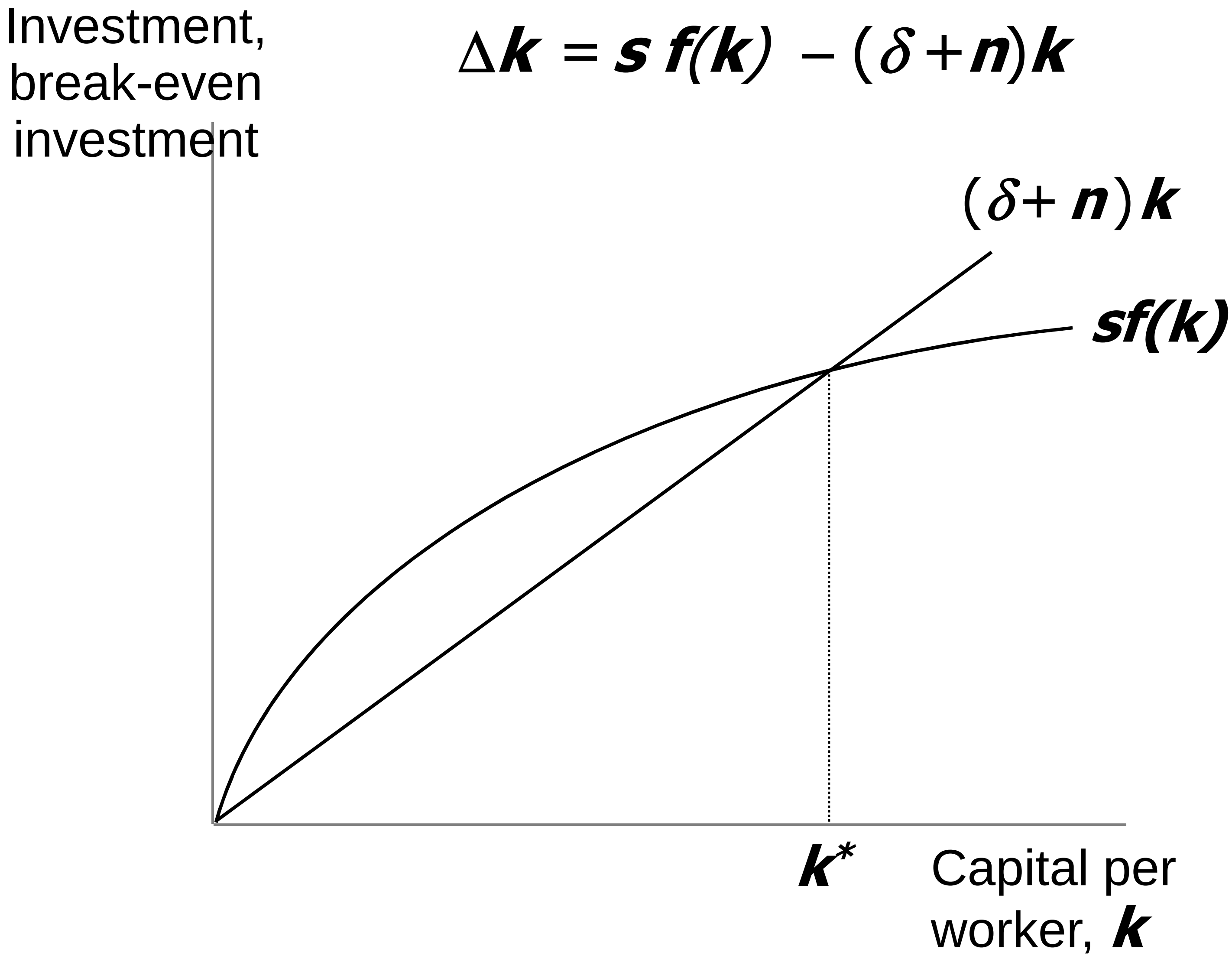
$$\Delta \boldsymbol{k} = \boldsymbol{s\, f(k)} - (\delta + \boldsymbol{n})\, \boldsymbol{k}$$

Actual (“new“) investment

Break-even investment necessary to keep k constant

7.3) Population Growth

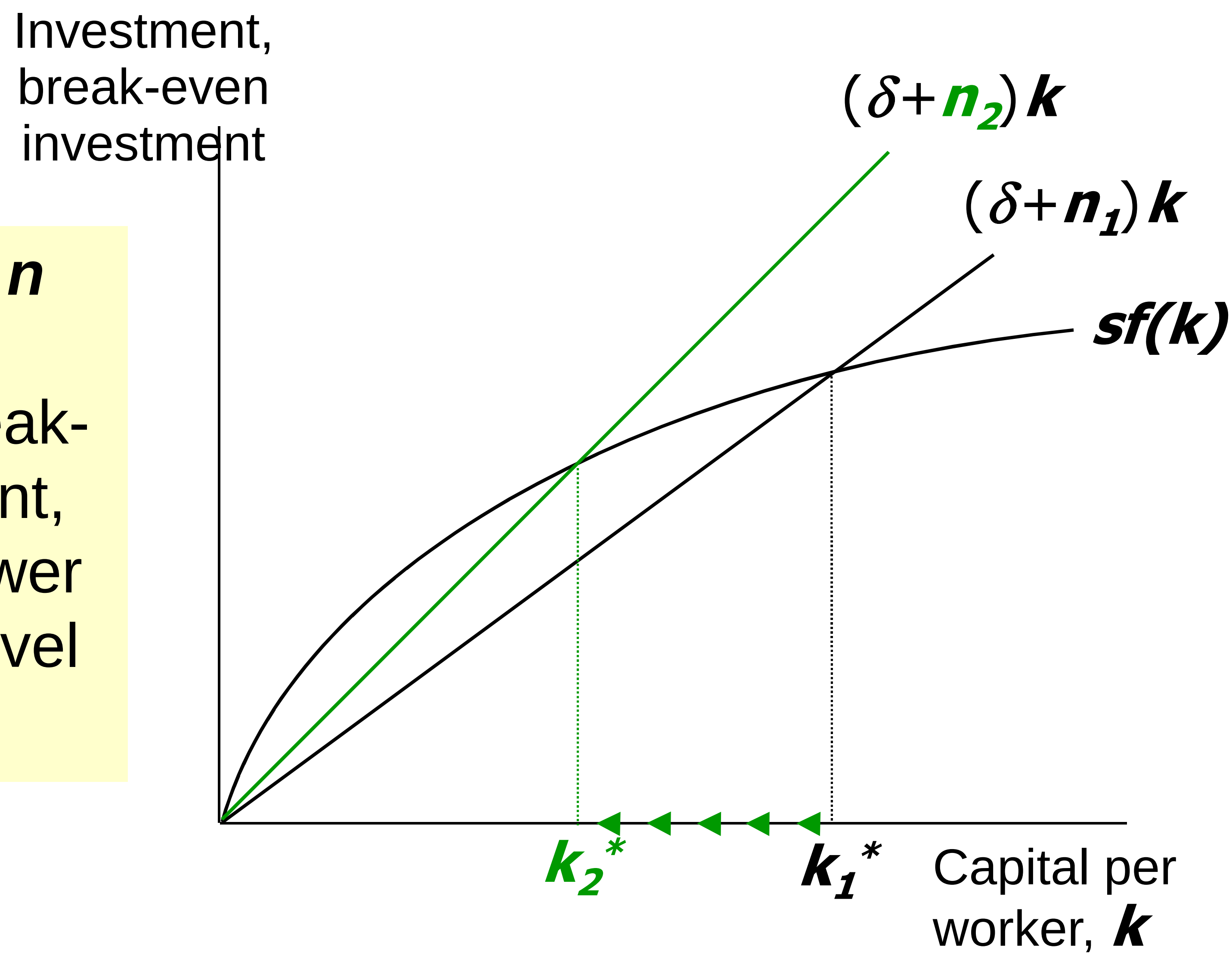
➔ SGM Diagram with Population Growth



7.3) Population Growth

➔ SGM and the Impact of Population Growth

An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.

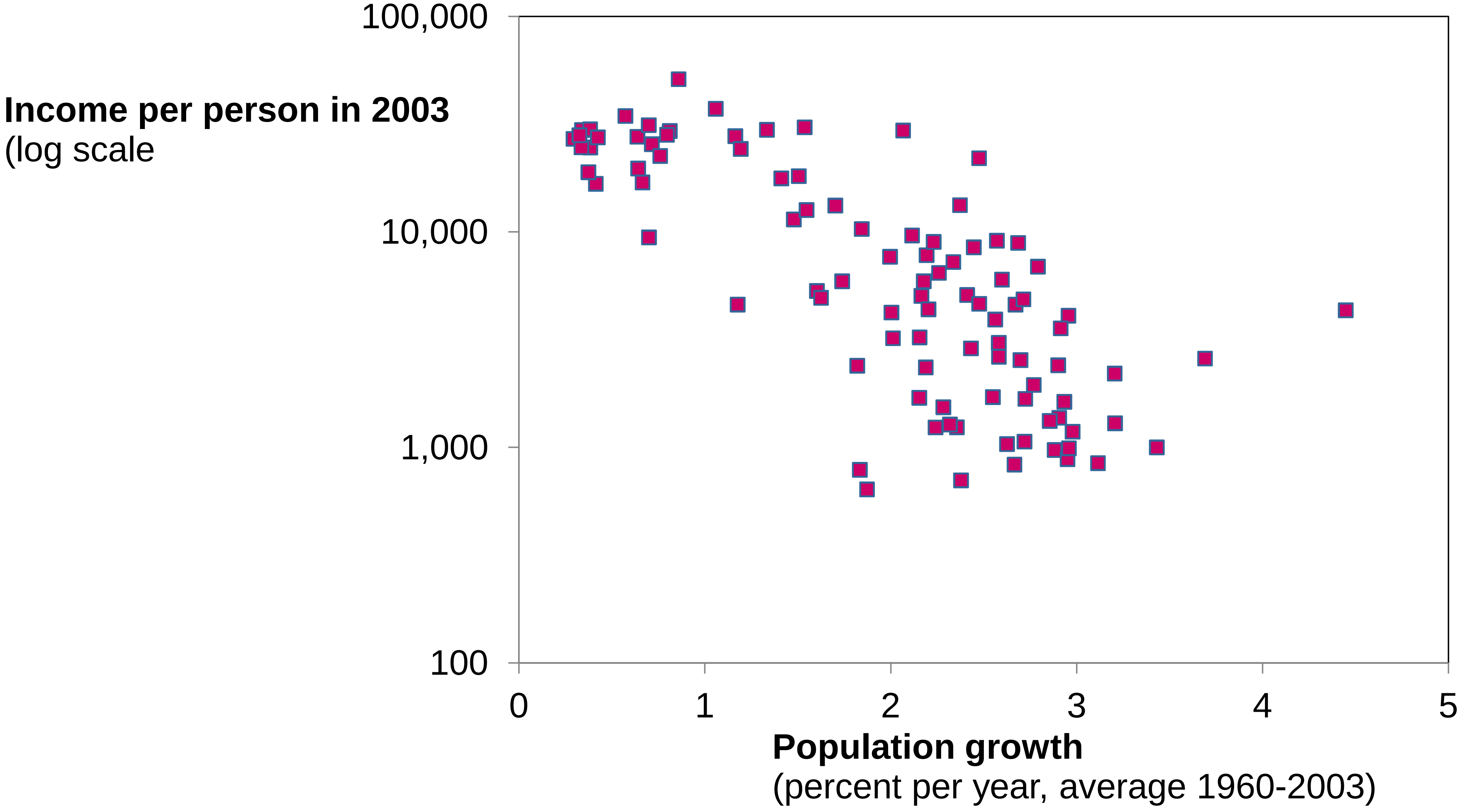


7.3) Population Growth

➔ SGM and Prediction Involving Pop. Growth

- Higher $\boldsymbol{n} \Rightarrow$ lower $\boldsymbol{k^*}$.
- And since $\boldsymbol{y} = f(\boldsymbol{k})$, lower $\boldsymbol{k^*} \Rightarrow$ lower $\boldsymbol{y^*}$.
- Thus, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

7.3) Population Growth
➔ International Evidence



7.3) Population Growth

➔ SGM and Golden Rule with Δn

- To find the Golden Rule capital stock, express $\boldsymbol{c}^*$ in terms of $\boldsymbol{k}^*$:

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n)\,k^*$$

- $\boldsymbol{c}^*$ is maximized when
  $$\text{MPK} = \delta + n$$
- or equivalently,
  $$\boxed{\text{MPK} - \delta = n}$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Economic Text:

→ The Growth of Growth Theory

1. From the perspective of the Solow model, why are “the efforts of policymakers to raise the rate of growth per head” ultimately futile?”
2. Which trick did Mr. Solow use to overcome the deficiency in his model that capital deepening, having diminishing returns, does not yield continuous economic growth?
3. What is meant with the phrase coined with respect to the Solow model “What it illuminated did not ultimately matter; and what really mattered it did little to illuminate”?
4. Tricky question for students with some knowledge in mathematical optimization: Why is the “topology of diminishing returns easy for economists to navigate”?
5. Which are the three points Mr Romer used to endogenize technological progress in his model?
6. What is meant with the last paragraph of the text?

Chapter Summary

1. The Solow growth model shows that, in the long run, a country's standard of living depends
- positively on its saving rate
- negatively on its population growth rate

2. An increase in the saving rate leads to
- higher output in the long run
- faster growth temporarily
- but not faster steady state growth

Chapter Summary (ctd.)

3. If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.

If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.