Chapter 14: A Dynamic Model of Aggregate Supply and Demand*

MACROECONOMICS
Seventh Edition
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* Slides based on Ron Cronovich's slides, adjusted for course in Macroeconomics for International Masters Program at the Wang Yanan Institute for Studies in Economics at Xiamen University.
This chapter introduces you to understanding:

- Elements of the DAS-DAD model
- How to solve the model
- How to use the model
- The model’s lessons for monetary policy
14.1) Elements of the Model

Introduction

• The dynamic model of aggregate demand and aggregate supply gives us more insight into how the economy works in the short run.

• It is a simplified version of a DSGE model, used in cutting-edge macroeconomic research.

(DSGE = Dynamic Stochastic General Equilibrium)
The dynamic model of aggregate demand and aggregate supply is built from familiar concepts, such as:

- the IS curve, which negatively relates the real interest rate and demand for goods & services
- the Phillips curve, which relates inflation to the gap between output and its natural level, expected inflation, and supply shocks
- adaptive expectations, a simple model of inflation expectations
14.1) Elements of the Model

- Difference between Dynamic/Static Model

• Instead of fixing the money supply, the central bank follows a monetary policy rule that adjusts interest rates when output or inflation change.

• The vertical axis of the DAD-DAS diagram measures the inflation rate, not the price level.

• Subsequent time periods are linked together: Changes in inflation in one period alter expectations of future inflation, which changes aggregate supply in future periods, which further alters inflation and inflation expectations.
14.1) Elements of the Model

Keeping Track of Time

- The subscript “\(t\)” denotes the time period, e.g.
  - \(Y_t\) = real GDP in period \(t\)
  - \(Y_{t-1}\) = real GDP in period \(t - 1\)
  - \(Y_{t+1}\) = real GDP in period \(t + 1\)

- We can think of time periods as years, E.g., if \(t = 2010\), then
  - \(Y_t = Y_{2010}\) = real GDP in 2010
  - \(Y_{t-1} = Y_{2009}\) = real GDP in 2009
  - \(Y_{t+1} = Y_{2011}\) = real GDP in 2011
14.1) Elements of the Model

• The model has five equations and five endogenous variables: output, inflation, the real interest rate, the nominal interest rate, and expected inflation.

• The equations may use different notation, but they are conceptually similar to things seen in course.

• The first equation is for output...
14.1) Elements of the Model

→ Output: Demand for Goods and Services

\[ Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t \]

- **output**
- **natural level of output**
- **real interest rate**

\[ \alpha > 0, \quad \rho > 0 \]

Negative relation between output and interest rate, same intuition as IS curve.
14.1) Elements of the Model

Real Interest Rate: The Fisher Equation

\[ r_t = i_t - E_t \pi_{t+1} \]

- *ex ante* (i.e. expected) real interest rate
- nominal interest rate
- expected inflation rate

\[ \pi_{t+1} = \text{increase in price level from period } t \text{ to } t+1, \text{ not known in period } t \]

\[ E_t \pi_{t+1} = \text{expectation, formed in period } t, \text{ of inflation from } t \text{ to } t+1 \]
14.1) Elements of the Model

Inflation: The Phillips Curve

\[ \pi_t = E_{t-1} \pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t \]

- \( \phi > 0 \) indicates how much inflation responds when output fluctuates around its natural level.
- supply shock, random and zero on average.
- previously expected inflation
- current inflation

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply
14.1) Elements of the Model

Expected Inflation: Adaptive Expectations

\[ E_t \pi_{t+1} = \pi_t \]

For simplicity, we assume people expect prices to continue rising at the current inflation rate.
14.1) Elements of the Model

Nominal Interest Rate: Taylor Rule

\[ i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t) \]

- *nominal interest rate*, set each period by the central bank
- *natural rate of interest*
- *central bank’s inflation target*

\[ \theta_\pi > 0, \theta_Y > 0 \]
14.1) Elements of the Model

Nominal Interest Rate: Taylor Rule

\[ i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t) \]

- \( i_t \) measures how much the central bank adjusts the interest rate when inflation deviates from its target.
- \( \pi_t \) measures how much the central bank adjusts the interest rate when output deviates from its natural rate.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 12/65
Economist John Taylor proposed a monetary policy rule very similar to ours:

\[ i_{ff} = \pi + 2 + 0.5(\pi - 2) - 0.5 (\text{GDP gap}) \]

where

- \( i_{ff} \) = nominal federal funds rate target
- GDP gap = 100 x \( \frac{\bar{Y} - Y}{\bar{Y}} \)
  = percent by which real GDP is below its natural rate

The Taylor Rule matches Fed policy fairly well....
14.1) Elements of the Model

→ Case Study: Taylor Rule

Taylor’s rule

actual Federal Funds rate

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 14/65
Learning Objectives

This chapter introduces you to understanding:

- Elements of the DAS-DAD model ✓
- How to solve the model
- How to use the model
- The model’s lessons for monetary policy
14.2) How to Solve the Model

The Model’s Variables and Parameters

Endogenous variables:

\[ Y_t = \text{Output} \]

\[ \pi_t = \text{Inflation} \]

\[ r_t = \text{Real interest rate} \]

\[ i_t = \text{Nominal interest rate} \]

\[ E_t \pi_{t+1} = \text{Expected inflation} \]
14.2) How to Solve the Model

The Model’s Variables and Parameters

Exogenous variables:

\( \bar{Y}_t \) = Natural level of output

\( \pi^*_t \) = Central bank’s target inflation rate

\( \varepsilon_t \) = Demand shock

\( \nu_t \) = Supply shock

Predetermined variable:

\( \pi_{t-1} \) = Previous period’s inflation
14.2) How to Solve the Model

The Model’s Variables and Parameters

Parameters:

\[ \alpha = \text{Responsiveness of demand to the real interest rate} \]

\[ \rho = \text{Natural rate of interest} \]

\[ \phi = \text{Responsiveness of inflation to output in the Phillips Curve} \]

\[ \theta_{\pi} = \text{Responsiveness of } i \text{ to inflation in the monetary-policy rule} \]

\[ \theta_{\gamma} = \text{Responsiveness of } i \text{ to output in the monetary-policy rule} \]
14.2) How to Solve the Model

The Model’s Long-Run Equilibrium

• The normal state around which the economy fluctuates.

• Two conditions required for long-run equilibrium:
  
  – There are no shocks: \( \varepsilon_t = \nu_t = 0 \)
  
  – Inflation is constant: \( \pi_{t-1} = \pi_t \)
14.2) How to Solve the Model

➔ The Model’s Long-Run Equilibrium

• Plugging the preceding conditions into the model’s five equations and using algebra yields these long-run values:

\[ Y_t = \bar{Y}_t \]

\[ r_t = \rho \]

\[ \pi_t = \pi^*_t \]

\[ E_t \pi_{t+1} = \pi^*_t \]

\[ i_t = \rho + \pi^*_t \]
14.2) How to Solve the Model

The Dynamic Aggregate Supply Curve

The DAS curve shows a relation between output and inflation that comes from the Phillips Curve and Adaptive Expectations:

\[ \pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t \quad (DAS) \]
14.2) How to Solve the Model

The Dynamic Aggregate Supply Curve

\[ \pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t \]

DAS slopes upward: high levels of output are associated with high inflation.

DAS shifts in response to changes in the natural level of output, previous inflation, and supply shocks.
To derive the DAD curve, we will combine four equations and then eliminate all the endogenous variables other than output and inflation.

Start with the demand for goods and services:

\[ Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \epsilon_t \]

\[ Y_t = \bar{Y}_t - \alpha(i_t - E_t \pi_{t+1} - \rho) + \epsilon_t \]

using the Fisher eq’n
14.2) How to Solve the Model

The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y}_t - \alpha (i_t - E_t \pi_{t+1} - \rho) + \varepsilon_t \]

using the expectations eq’n

\[ Y_t = \bar{Y}_t - \alpha (i_t - \pi_t - \rho) + \varepsilon_t \]

using monetary policy rule

\[ Y_t = \bar{Y}_t - \alpha \left[ \frac{\pi_t}{\pi_t*} + \rho + \theta_\pi (\pi_t - \pi_t*) + \theta_Y (Y_t - \bar{Y}_t) \right] - \pi_t - \rho + \varepsilon_t \]

\[ Y_t = \bar{Y}_t - \alpha \left[ \theta_\pi (\pi_t - \pi_t*) + \theta_Y (Y_t - \bar{Y}_t) \right] + \varepsilon_t \]
14.2) How to Solve the Model

The Dynamic Aggregate Demand Curve

result from previous slide

\[ Y_t = \bar{Y}_t - \alpha [\theta_{\pi} (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t)] + \epsilon_t \]

combine like terms, solve for \( Y \)

\[ Y_t = \bar{Y}_t - A(\pi_t - \pi_t^*) + B \epsilon_t, \quad \text{(DAD)} \]

where

\[ A = \frac{\alpha \theta_{\pi}}{1 + \alpha \theta_Y} > 0, \quad B = \frac{1}{1 + \alpha \theta_Y} > 0 \]
14.2) How to Solve the Model

The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y}_t - A(\pi_t - \pi_t^*) + B\varepsilon_t \]

DAD slopes downward:
When inflation rises, the central bank raises the real interest rate, reducing the demand for goods & services.

DAD shifts in response to changes in the natural level of output, the inflation target, and demand shocks.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 26/65
14.2) How to Solve the Model

Short-run Equilibrium in the Model

The combination of Output and Inflation that solve the DAD and DAS equations, given full-employment output, target inflation, lagged inflation, and the shock terms.

**DAD:**

\[ Y_t = \bar{Y}_t - A(\pi_t - \pi_t^*) + B\varepsilon_t \]

**DAS:**

\[ \pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t \]
In each period, the intersection of DAD and DAS determines the short-run eq’m values of inflation and output.

In the eq’m shown here at A, output is below its natural level.
Learning Objectives

This chapter introduces you to understanding:

- Elements of the DAS-DAD model ✓
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14.3) How to Use the Model

→ Effect of Change in Long-Run Growth

Period: initial eq'm at $A$.

$\pi_t$: $DAS_t$

Period $t + 1$: $Y_{t+1}$

$DAS_t$ shifts because economy can produce more g&s.

New eq'm at $B$, income grows but inflation remains stable.

$\pi_t = \pi$

$\pi_{t+1}$

$Y$: $DAD_t$

$DAD_{t+1}$

PeriDAS shifts initially because higher income raises demand for g&s.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 30/65
14.3) How to Use the Model

Effect of Shock to Aggregate Supply

Period $t - 1$:
- Initial eq’m at $A$

Period $t$:
- Supply shock ($\nu > 0$) shifts DAS upward; inflation rises, central bank responds by raising real interest rate, output falls.

Period $t + 1$:
- Supply shock is over ($\nu = 0$) but DAS does not return to its initial position due to higher inflation expectations. DAS moves downward, output rises.

Period $t + 2$:
- As inflation falls, inflation expectations fall, DAS moves downward, output rises. This process continues until output returns to its natural rate. LR eq’m at $A$.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply
### 14.3) How to Use the Model

**Shock to AS: Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{Y}_t$</td>
<td>100</td>
</tr>
<tr>
<td>$\pi_t^*$</td>
<td>2.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2.0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\theta_\pi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta_Y$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Thus, we can interpret $Y_t - \bar{Y}_t$ as the percentage deviation of output from its natural level. Central bank’s inflation target is 2 percent.

A 1-percentage-point increase in the real interest rate reduces output demand by 1 percent of its natural level.

These values are from the Taylor Rule, which approximates the actual behavior of the Federal Reserve.

The following graphs are called *impulse response functions*. They show the “response” of the endogenous variables to the “impulse,” i.e., the shock.
14.3) How to Use the Model

Shock to AS: Dynamic Response

A one-period supply shock affects output for many periods.
Because inflation expectations adjust slowly, actual inflation remains high for many periods.
14.3) How to Use the Model

Shock to AS: Dynamic Response

The behavior of the nominal interest rate depends on the interest rule of the central bank.
The real interest rate depends on the nominal interest rate and inflation.
14.3) How to Use the Model

Shock to Aggregate Demand

Period $t$:
- $\pi_{t-1}$: Initial eq’m at A
- $DAS_t + 1$: Higher inflation

Period $t+1$:
- Positive demand shock ($\varepsilon > 0$) shifts AD to the right; output and inflation rise.

Periods $t+2$ to $t+4$:
- Higher inflation in previous period raises inflation expectations, shifting DAS up. Inflation rises, output falls.

Period $t+5$:
- DAS is higher due to higher inflation in preceding period, but demand shock ends and DAD returns to its initial position. Eq’m at A.

Periods $t+6$ and higher:
- DAS gradually shifts down as inflation and inflation expectations fall, economy gradually recovers until reaching LR eq’m at A.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 37/65
The demand shock raises output for five periods. When the shock ends, output falls below its natural level, and recovers gradually.
The demand shock causes inflation to rise. When the shock ends, inflation gradually falls toward its initial level.
The behavior of the nominal interest rate depends on the monetary policy rule.
The demand shock raises the real interest rate. After the shock ends, the real interest rate falls and approaches its initial level.
Learning Objectives

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Period $t$: target inflation rate $\pi^* = 2\%$, initial eq'm at $\pi_{t-1} = 2\%$

Period $t$: Central bank lowers target to $\pi^* = 1\%$, raises $\pi_t$

Period $t+1$: The fall in $\pi_t$ reduced inflation expectations for $t+1$, shifting $\pi_{DAS_{t-1}, t}$, $\pi_{Y_{A}}$, $\pi_{Y_{B}}$ $\pi_{DAD_{t-1}}$, $\pi_{DAD_{t,t+1}}$,…

Subsequent periods: This process continues until output returns to its natural rate and inflation reaches its new target.

Chapter 14: A Dynamic Model of Aggregate Demand and Aggregate Supply 43/65
Reducing the target inflation rate causes output to fall below its natural level for a while. Output recovers gradually.
Because expectations adjust slowly, it takes many periods for inflation to reach the new target.
The central bank increases the nominal interest rate to achieve its new inflation target. As the inflation and real interest rates fall, the nominal rate falls.
The increase in the nominal interest rate increases the real interest rate. As inflation falls, the real interest rate gradually returns to its natural rate.
Learning Objectives

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14.4) Lessons for Monetary Pol.

→ Output Variability vs. Inflation Variability

- A supply shock reduces output (bad) and raises inflation (also bad).
- The central bank faces a tradeoff between these “bads” – it can reduce the effect on output, but only by tolerating an increase in the effect on inflation....
14.4) Lessons for Monetary Pol.

Output Variability vs. Inflation Variability

CASE 1: $\theta_\pi$ is large, $\theta_Y$ is small

A supply shock shifts DAS up.

In this case, a small change in inflation has a large effect on output, so DAD is relatively flat.

$$Y_t = \bar{Y}_t - \frac{\alpha \theta_\pi}{1 + \alpha \theta_Y} (\pi_t - \pi_t^*) + \frac{1}{1 + \alpha \theta_Y} \varepsilon_t$$

(DAD)
CASE 2: $\theta_\pi$ is small, $\theta_Y$ is large

In this case, a large change in inflation has only a small effect on output, so DAD is relatively steep.

\[
Y_t = \bar{Y}_t - \frac{\alpha \theta_\pi}{1 + \alpha \theta_Y} (\pi_t - \pi^*_t) + \frac{1}{1 + \alpha \theta_Y} \varepsilon_t \quad \text{(DAD)}
\]
14.4) Lessons for Monetary Pol.

⇒ Application: The Taylor Principle

- **The Taylor Principle** (named after John Taylor): The proposition that a central bank should respond to an increase in inflation with an even greater increase in the nominal interest rate (so that the real interest rate rises).

  *i.e.*, central bank should set $\theta_\pi > 0$.

- Otherwise, DAD will slope upward, economy may be unstable, and inflation may spiral out of control.
14.4) Lessons for Monetary Pol.

Application: The Taylor Principle

\[ Y_t = \bar{Y}_t - \frac{\alpha \theta_\pi}{1 + \alpha \theta_y} (\pi_t - \pi_t^*) + \frac{1}{1 + \alpha \theta_y} \varepsilon_t \quad (DAD) \]

\[ i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_y (Y_t - \bar{Y}_t) \quad (MP \ rule) \]

If \( \theta_\pi > 0 \):

- When inflation rises, the central bank increases the nominal interest rate even more, which increases the real interest rate and reduces the demand for goods & services.

- DAD has a negative slope.
14.4) Lessons for Monetary Pol.

Application: The Taylor Principle

\[ Y_t = \overline{Y}_t - \frac{\alpha \theta_{\pi}}{1 + \alpha \theta_Y} (\pi_t - \pi_t^*) + \frac{1}{1 + \alpha \theta_Y} \varepsilon_t \quad (DAD) \]

\[ i_t = \pi_t + \rho + \theta_{\pi} (\pi_t - \pi_t^*) + \theta_Y (Y_t - \overline{Y}_t) \quad (MP \ rule) \]

If \( \theta_{\pi} < 0 \):

- When inflation rises, the central bank increases the nominal interest rate by a smaller amount. The real interest rate falls, which increases the demand for goods & services.

- DAD has a positive slope.
14.4) Lessons for Monetary Pol.

Application: The Taylor Principle
14.4) Lessons for Monetary Pol.

Application: The Taylor Principle

- If DAD is upward-sloping and steeper than DAS, then the economy is unstable: output will not return to its natural level, and inflation will spiral upward (for positive demand shocks) or downward (for negative ones).

- Estimations of $\theta_\pi$ from published research:
  - $\theta_\pi = -0.14$ from 1960-78, before Paul Volcker became Fed chairman. Inflation was high during this time, especially during the 1970s.
  - $\theta_\pi = 0.72$ during the Volcker and Greenspan years. Inflation was much lower during these years.
Chapter Summary

• The DAD-DAS model combines five relationships: an IS-curve-like equation of the goods market, the Fisher equation, a Phillips curve equation, an equation for expected inflation, and a monetary policy rule.
• The long-run equilibrium of the model is classical. Output and the real interest rate are at their natural levels, independent of monetary policy. The central bank’s inflation target determines inflation, expected inflation, and the nominal interest rate.
Chapter Summary

• The DAD-DAS model can be used to determine the immediate impact of any shock on the economy, and can be used to trace out the effects of the shock over time.
• The parameters of the monetary policy rule influence the slope of the DAD curve, so they determine whether a supply shock has a greater effect on output or inflation. Thus, the central bank faces a tradeoff between output variability and inflation variability.
• The DAD-DAS model assumes that the Taylor Principle holds, i.e. that the central bank responds to an increase in inflation by raising the real interest rate. Otherwise, the economy may become unstable and inflation may spiral out of control.