Special Topics in Applied Econometrics
1) LINEAR PANEL MODELS

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1) Linear Panel Models:
$\rightarrow$ Agenda

- $\Rightarrow$ Introduction
- The Pooled Model
- Unobserved Effects Panel Data Models

General linear model

$$y_{it} = \alpha_{it} + \mathbf{x}_{it}\beta_{it} + u_{it} \tag{1}$$

where $i = 1, 2, \cdots, N$ and $t = 1, 2, \cdots, T$

- Coefficients and slope vary over individual and time
- Model too general and not estimable

1) Linear Panel Models:
$\rightarrow$ Agenda

- Introduction
- $\Rightarrow$ The Pooled Model
- Unobserved Effects Panel Data Models

$$y_{it} = \alpha + \mathbf{x}_{it}\beta + u_{it} \tag{2}$$

Assumptions:

- $E(\mathbf{x}_t^{'} u_t) = \mathbf{0}, t = 1, 2, \cdots, T$
  'Regressors are contemporaneously uncorrelated with the error term'
- $rank[\sum_{t=1}^{T} E(\mathbf{x}_t^{'}\mathbf{x}_t)] = k$
  'Regressors are not perfectly linear dependent'
- $E(u_t^2 \mathbf{x}_t^{'}\mathbf{x}_t) = \sigma^2 E(\mathbf{x}_t^{'}\mathbf{x}_t),\ t = 1, 2, \cdots, T;\ \sigma^2 = E(u_t^2)$
  'Homoscedasticity'
- $E(u_t u_s \mathbf{x}_t^{'}\mathbf{x}_s) = \mathbf{0},\ t \neq s;\ t, s = 1, 2, \cdots, T$
  'Conditional covariance of errors across time periods are zero'

1.2) The Pooled Model

The pooled ordinary least squre (OLS) estimator is given by

$$\hat{\theta} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} \tag{3}$$

where $\mathbf{X}$ and $\mathbf{y}$ are the stacked explanatory variables x_{it}^k and dependend variable y_{it} over time, respectively:

1.2) The Pooled Model

$$\mathbf{y} = \begin{bmatrix} y_{11} \\ \vdots \\ y_{1T} \\ y_{21} \\ \vdots \\ y_{2T} \\ \vdots \\ y_{N1} \\ \vdots \\ y_{NT} \end{bmatrix}_{\text{NTx1}} \qquad \hat{\theta} = \begin{bmatrix} \hat{\alpha} \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_K \end{bmatrix}_{\text{K+1x1}} \qquad \mathbf{X} = \begin{bmatrix} 1 & x^1_{11} & \cdots & x^K_{11} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x^1_{1T} & \cdots & x^K_{1T} \\ 1 & x^1_{21} & \cdots & x^K_{21} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x^1_{2T} & \cdots & x^K_{2T} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x^1_{N1} & \cdots & x^K_{N1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x^1_{NT} & \cdots & x^K_{NT} \end{bmatrix}_{\text{NTxK+1}}$$

where

$k = 1, \cdots, K$ denote different regressors

$i = 1, \cdots, N$ denote the cross-section dimension

$t = 1, \cdots, T$ denote the time dimension

1.2) The Pooled Model

The estimator for the asymptotic variance of $\hat{\beta}$ is given by

$$Avar\left(\hat{\theta}\right) = \hat{\sigma}^2 \left(\mathbf{X}'\mathbf{X}\right)^{-1} \tag{4}$$

where $\hat{\sigma}^2$ is the usual OLS variance estimator from the pooled regression $\frac{1}{NT-K-1}\hat{\mathbf{u}}'\hat{\mathbf{u}}$ with

$$\hat{u} = \begin{bmatrix} \hat{u}_{11} \\ \vdots \\ \hat{u}_{1T} \\ \vdots \\ \hat{u}_{N1} \\ \vdots \\ \hat{u}_{NT} \end{bmatrix}$$

- A heteroscedasticity and serial correlation robust estimate of $\hat{\beta}$ can be obtained as

$$Avar(\hat{\theta}) = (\sum_{i=1}^{N} \mathbf{X}_i'\mathbf{X})^{-1}(\sum_{i=1}^{N} \mathbf{X}_i'\hat{\mathbf{u}}_i\hat{\mathbf{u}}_i'\mathbf{X})(\sum_{i=1}^{N} \mathbf{X}_i'\mathbf{X})^{-1} \quad (5)$$

where the residuals $\hat{\mathbf{u}}_i$ are the $T \times 1$ pooled OLS residuals for cross-section observation i:

$$\hat{\mathbf{u}}_i = \begin{bmatrix} \hat{u}_{11} \\ \vdots \\ \hat{u}_{1T} \end{bmatrix}_{\text{NTx1}}$$

- Whether a robust variance estimator is necessary can be infered via testing for serial correlation and heteroscedasticity

1.2) The Pooled Model
$\Rightarrow$ Testing for Serial Correlation

- Estimate Equation (2) by pooled OLS and obtain the errors $\hat{u}_{it}$
- Estimate Equation (2), including the lagged errors as additional explanatory variable in $\mathbf{x}_{it}$

$$y_{it} = \tilde{\alpha} + \tilde{\mathbf{x}}_{it}\tilde{\beta} + \tilde{u}_{it} \tag{6}$$

where $\tilde{\mathbf{x}}_{it}$ contains the lagged error term

- Compute heteroscedasticity robust variance for $\tilde{\theta} = \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{bmatrix}$

$$Avar(\hat{\tilde{\theta}}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Psi}}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \tag{7}$$

where $\hat{\mathbf{\Psi}} = diag(\hat{\tilde{u}}_{11}^2, \hat{\tilde{u}}_{12}^2, ..., \hat{\tilde{u}}_{1T}^2, \hat{\tilde{u}}_{21}^2, ..., ..., \hat{\tilde{u}}_{NT}^2)$

- If the usual t-statistic for the coefficient of the lagged $\hat{u}_{it}$ is significant, serial correlation is present

1.2) The Pooled Model
$\Rightarrow$ Testing for Heteroscedasticity

- The null hypothesis (H_0) can be stated as $E(u_t^2|\mathbf{x}_t) = \sigma^2,\ t = 1, 2, ..., T$
- Obtain R_c^2 from the POLS regression of $\hat{u}_{it}^2$ on a constant and $\mathbf{h}_{it};\ t = 1, ..., T;\ i = 1, ..., N$:

$$\hat{u}_{it}^2 = const + \mathbf{h}_{it}\tau + \epsilon_{it} \tag{8}$$

where:

  - $\mathbf{h}_{it}$ contains elements of $\mathbf{x}_{it}$, as well as squares and cross-products of elements of $\mathbf{x}_{it}$
  - $R_c^2 = 1 - \frac{\hat{\epsilon}'\hat{\epsilon}}{\hat{\hat{\epsilon}}'\hat{\hat{\epsilon}}}$ with $\hat{\epsilon}$ the stacked error terms from Equation (8) and $\hat{\hat{\epsilon}}$ the stacked error terms from regressing $\hat{u}_{it}^2$ on a constant only
- Under H_0 the test statistic NTR_c^2 has a χ_Q^2 distribution, with Q the number of elements in $\mathbf{h}_{it}$

1.2) The Pooled Model
$\Rightarrow$ Digression: The Solow Growth Model

- Solow (1956) obtained Nobel prize for model
- Simple framework that helps investigating the proximate causes and mechanics of the process of economic growth
- Model contains abstract representation of a complex economy. Supply of goods based on classical production function: $Y = F(K, L)$
- One theoretical implication of the model: 'convergence'
- Solow model can be used to investigate economic growth over time
- In particular, regression analyses have been extensively used to confront theory with empirics
- Barro-type regressions to investigate 'convergence' and the causes of economic growth

1.2) The Pooled Model
⇒ Digression: The Solow Growth Model (ctd.)

- The basic Solow model with constant population growth and labor augmenting technological change with a Cobb-Douglas production function can be used to derive the following equation [see, for example, Acemoglu (2008)]

$$\frac{y_{it} - y_{i,t-1}}{y_{i,t-1}} \approx g - \tau(log\ y_{i,t-1} - log\ y^*_{i,t-1})$$

  where

  - y is output
  - y^* is equilibrium output to which economies converge
  - g is technological progress
  - $\tau = (1 - \alpha)(\delta + g + n)$, with α a parameter from the Cobb-Douglas production function, δ the depreciation of capital, and n the population growth

1.2) The Pooled Model
⇒ Digression: The Solow Growth Model (ctd.)

- Assuming y^* constant, the equation can be put into the following estimable framework

$$\frac{y_{it} - y_{i,t-1}}{y_{i,t-1}} = \tilde{y}_{it} \approx c + \tau log\ y_{i,t-1} + \epsilon_{it}$$

where $c = g + log\ \tau y^*$

- In case of convergence, τ should be smaller than zero
- This model implication can be empirically tested via estimating the above equation

1.2) The Pooled Model
⇒ Digression: Introduction to Matlab

Introduction to Matlab

1.2) The Pooled Model
⇒ HANDS-ON

Hands-On 1

1.2) The Pooled Model
⇒ Digression: The Solow Growth Model (ctd.)

- The estimated growth equation is usually refered to as 'unconditional convergence' (Sala-i-Martin, 1992), that is, countries converge regardless of differences in individual characteristics and policies
- This may be too demanding, since it implies that the income gap between any two countries irrespective of their institutional environment, investment behavior, policies etc. should shrink over time
- If countries differ with respect to these factors, the Solow model predicts that they converge to different levels y^*
- A more appropriate regression equation may thus take the form

$$\tilde{y}_{it} \approx c_i + \tau log\ y_{i,t-1} + \epsilon_{it}$$

1) Linear Panel Models: $\rightarrow$ Agenda

- Introduction
- The Pooled Model
- $\Rightarrow$ Unobserved Effects Panel Data Models

- Key assumption in previous sub-section: $E(\mathbf{x}_t' u_t) = \mathbf{0};\ t = 1, 2, ..., T$
- This assumption is very likely to be violated in the model from Equation (2)
- Panel data models explicitely containing a time constant and an unobserved effect can solve/alleviate the omitted variables problem
- Key issue: is the unobserved effect uncorrelated with the explanatory variables?

1.3) Unobserved Effects Panel Data Models
⇒ Introduction (ctd.)

$$y_{it} = c_i + \mathbf{x}_{it}\beta + u_{it} \tag{9}$$

- Implicit assumption in Equation (9):
  Each cross-section features a time constant unobserved variable ('unobserved effect')
- The unobserved effect captures features of individual cross-sections that are given and do not change over time (e.g. socio-cultural factors such as religion)
- To consistently estimate the parameter vector of interest, β, it is crucial whether the unobserved effect, c_i, is (Fixed Effects model) or is not (Random Effects model) correlated with the vector of explanatory variables, $\mathbf{x}_{it}$ for all t

$$y_{it} = \mathbf{x}_{it}'\beta + v_{it} \tag{10}$$

where $v_{it} = u_{it} + c_i$

Under the following assumptions the coefficient vector of interest can be estimated consistently and efficiently by feasible generalized least squares (FGLS) within an RE framework:

- $E(u_{it}|\mathbf{x}_i, c_i) = 0$ for $t = 1, ..., T$; with $\mathbf{x}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ...\mathbf{x}_{iT})$
  'Strict exogeneity of the explanatory variables and the unobserved effect'
- $E(c_i|\mathbf{x}_i) = E(c_i) = 0$
  'Orthogonality between c_i and $\mathbf{X}_i$'. The second part of this assumption is without loss of generality if $\mathbf{x}_{it}$ contains an intercept

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Random Effects (RE) Methods

- *rank* $E(\mathbf{X}_i'\mathbf{\Omega}^{-1}\mathbf{X}_i) = K$
  'Regressors are not perfectly linear dependent'
  where the elements of $\mathbf{\Omega}$ are determined by two further assumptions on the idiosyncratic errors:
  - $E(u_{it}^2) = \sigma_u^2;\ t = 1, 2, ..., T$
    'Constant unconditional variance across t'
  - $E(u_{it}u_{is}) = 0;\ \forall t \neq s$
    'Idiosyncratic errors are serially uncorrelated'
- $E(\mathbf{u}_i\mathbf{u}_i'|\mathbf{x}_i, c_i) = \sigma_u^2\mathbf{I}_T$
  Orthogonality of outer product of idiosyncratic errors with respect to $\mathbf{x}_i$ and c_i implies the previous assumptions (constant variance and serial uncorrelatedness)
- $E(c_i^2|\mathbf{x}_i) = \sigma_c^2$
  'Homoscedasticity of the unobserved effect'

- For the *FGLS* procedure, define $\sigma_v^2 = \sigma_c^2 + \sigma_u^2$
- If we have consistent estimators of σ_u^2 and σ_c^2 we can form

$$\hat{\mathbf{\Omega}} = \hat{\sigma}_u^2 \mathbf{I}_T + \hat{\sigma}_c^2 \iota_T \iota_T'$$

where ι_T is a vector of ones of dimension $T \times 1$

- The FGLS-RE estimator is given by

$$\hat{\beta}_{RE} = (\sum_{i=1}^{N} \mathbf{X}_i' \hat{\mathbf{\Omega}} \mathbf{X}_i)^{-1} (\sum_{i=1}^{N} \mathbf{X}_i' \hat{\mathbf{\Omega}} \mathbf{y}_i) \tag{11}$$

- The asymptotic variance $Avar(\hat{\beta}_{RE})$ is given by

$$(\sum_{i=1}^{N} \mathbf{X}_i' \hat{\mathbf{\Omega}}^{-1} \mathbf{X}_i)^{-1} \sum_{i=1}^{N} \mathbf{X}_i' \hat{\mathbf{\Omega}}^{-1} \hat{\mathbf{v}}_i \hat{\mathbf{v}}_i' \hat{\mathbf{\Omega}}^{-1} \mathbf{X}_i (\sum_{i=1}^{N} \mathbf{X}_i' \hat{\mathbf{\Omega}}^{-1} \mathbf{X}_i)^{-1} \tag{12}$$

- To implement the RE procedure, we need $\hat{\sigma}_c^2$ and $\hat{\sigma}_u^2$
- Given the previous assumptions we have $\sigma_v^2 = \sigma_c^2 + \sigma_u^2$ with $\sigma_v^2 = T^{-1} \sum_{t=1}^{T} E(v_{it}^2)\ \forall i$
- Estimation procedure:
  1. Estimate Equation (10) using pooled OLS
  2. Obtain the pooled OLS residuals, $\hat{\hat{v}}_{it}$
  3. Compute consistent estimator of σ_v^2: $\sigma_v^2 = \frac{1}{NT-K} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\hat{v}}_{it}^2$
  4. Compute consistent estimator of $\hat{\sigma}_c^2$: $\hat{\sigma}_c^2 = \frac{1}{NT(T-1)/2-K} \sum_{i=1}^{N} \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \hat{\hat{v}}_{it} \hat{\hat{v}}_{is}$
  5. Form $\hat{\mathbf{\Omega}}$ and obtain the parameter vector of interest using Equation (11)
  6. Use Equation (12) to compute $Avar(\hat{\beta}_{RE})$

- If the idiosyncratic errors are heteroscedastic and serially correlated across t, a more general estimator $\hat{\mathbf{\Omega}}$ can be used:

$$\hat{\mathbf{\Omega}} = N^{-1} \sum_{i=1}^{N} \hat{\hat{\mathbf{v}}}_i \hat{\hat{\mathbf{v}}}_i'$$

  where the $\hat{\hat{\mathbf{v}}}_i$ are the pooled OLS residuals

- If N is not several times larger than T, an unrestricted FGLS features undesireable finite sample properties whereas the RE aproach only requires estimation of two variance parameters

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Random Effects (RE) Methods

- If the RE assumptions hold but the model does actually not contain an unobserved effect, pooled OLS is efficient
- To test $H_0 : \sigma_c^2 = 0$, the following asymptotically standard normally distributed test statistic can be used:

$$TS = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \hat{v}_{it} \hat{v}_{is}}{[\sum_{i=1}^{N} (\sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \hat{v}_{it} \hat{v}_{is})^2]^{\frac{1}{2}}} \tag{13}$$

1.3) Unobserved Effects Panel Data Models
⇒ Digression: Manifesto for a Growth Econometrics

DISCUSSION

1.3) Unobserved Effects Panel Data Models ⇒ HANDS-ON

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1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods

$$y_{it} = \mathbf{x}_{it}\beta + c_i + u_{it} \tag{14}$$

- Assuming that c_i and $\mathbf{x}_{it}$ are uncorrelated, the RE approach puts c_i into the error term
- FE methods allow for c_i to be arbitrarily correlated with $\mathbf{x}_{it}$
- This robustness comes at a price: we cannot include time constant factors in $\mathbf{x}_{it}$

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods (ctd.)

To allow for estimation within an FE framework the following assumptions need to hold:

- $E(u_{it}|\mathbf{x}_i, c_i) = 0, \ \text{for } t = 1, 2, ..., T$; with $\mathbf{x}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ...\mathbf{x}_{iT})$
  'Strict exogeneity of the explanatory variables and the unobserved effect'
- $rank(\sum_{t=1}^{T} E(\ddot{\mathbf{x}}_{it}'\ddot{\mathbf{x}}_{it})) = rank(\sum_{t=1}^{T} E(\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i)) = K$
  'Regressors are not perfectly linear dependent'
  with $\ddot{\mathbf{x}}_{it}$ the time demeaned explanatory variables: $\ddot{\mathbf{x}}_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i$
- $E(\mathbf{u}_i\mathbf{u}_i'|\mathbf{x}_i, c_i) = \sigma_u^2\mathbf{I}_T$
  'Homoscedasticity and serial uncorrelatedness of the idiosyncratic errors'

1.3) Unobserved Effects Panel Data Models
⇒ Fixed effects (FE) Methods (ctd.)

- Under the given assumptions the FE model can be estimated consistently and efficiently by applying pooled OLS after a fixed effects transformation of Equation (14)
- The fixed effects transformation removes the unobserved effect from Equation (14)
- Alternative transformations are available, for example, a first-differencing transformation

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods (ctd.)

Estimation of the FE model proceeds in three steps:

1. Average Equation (14) over $t = 1, 2, ..., T$

$$\bar{y}_i = \bar{\mathbf{x}}_i \beta + c_i + \bar{u}_i \tag{15}$$

   with $\bar{y}_i = T^{-1} \sum_{t=1}^{T} y_{it}$

2. Subtract Equation (15) from Equation (14) on the FE transformed equation to obtain the FE transformed equation

$$y_{it} - \bar{y}_i = (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)\beta + u_{it} - \bar{u}_i$$

   or

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}\beta + \ddot{u}_{it}; \;\; t = 1, 2, ..., T$$

3. Use the pooled OLS estimator from Equation (3) on the transformed equation to obtain $\hat{\beta}_{FE}$

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods (ctd.)

To obtain the asymptotic variance of the estimated parameter vector, $Avar(\hat{\beta}_{FE})$ one proceed in two steps:

- Estimate the variance of the idiosyncratic errors

$$\hat{\sigma}_u^2 = \frac{1}{N(T-1)-K}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{u}_{it}^2$$

  where $\hat{u}_{it} = \ddot{y}_{it} - \ddot{\mathbf{x}}_{it}'\beta_{FE};\;\; t = 1, 2, ..., T;\;\; i = 1, 2, ..., N$

- The variance-covariance matrix is given by

$$Avar(\hat{\beta}_{FE}) = \sigma_u^2(\sum_{i=1}^{N}\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i)^{-1} \qquad (16)$$

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods (ctd.)

If serial correlation is present in the idiosyncratic errors the variance matrix estimator in Equation (16) will be improper.
To test the idiosyncratic errors for serial correlation one proceeds as follows:

- Run the regression $\hat{u}_{iT}$ on $\hat{u}_{i,T-1}$; $i = 1, 2, ..., N$
  with $\hat{u}_{iT}, \hat{u}_{i,T-1}$ the last two time periods of cross-section i
- Test H_0 : 'No serial correlation:' $\delta = -\frac{1}{T-1}$ (see Wooldridge (2004), p. 275) where δ is the coefficient of $\hat{u}_{i,T-1}$. Under the previous assumptions the usual t-statistic is asymptotically normally distributed

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ Fixed effects (FE) Methods (ctd.)

- If serial correlation is present, the asymptotic variance estimator needs to be adjusted. The robust variance estimator of $\hat{\beta}_{FE}$ is

$$Avar(\hat{\beta}_{FE}) = (\ddot{\mathbf{X}}'\ddot{\mathbf{X}})^{-1}(\sum_{i=1}^{N} \ddot{\mathbf{X}}_i'\hat{\mathbf{u}}_i\hat{\mathbf{u}}_i'\ddot{\mathbf{X}}_i)(\ddot{\mathbf{X}}'\ddot{\mathbf{X}})^{-1} \quad (17)$$

where $\hat{\mathbf{u}}_i = \mathbf{y}_i - \ddot{\mathbf{X}}'\hat{\beta}_{FE}$

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Fixed effects (FE) Methods (ctd.)

Alternatively to computing a robust variance estimator of $\hat{\beta}_{FE}$ we can relax two previous assumptions:

- Instead of $E(\mathbf{u}_i\mathbf{u}_i'|\mathbf{X}_i, c_i) = \sigma_u^2\mathbf{I}_T$ we assume $E(\mathbf{u}_i\mathbf{u}_i'|\mathbf{X}_i, c_i) = \mathbf{\Lambda}$, with $\mathbf{\Lambda}$
  - a $T \times T$ positive definite matrix
  - an unrestricted, constant covariance matrix
- Instead of $rank(E(\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i)) = K$ we assume $rank(E(\ddot{\mathbf{X}}_i'\mathbf{\Omega}\ddot{\mathbf{X}}_i)) = K$, with $\mathbf{\Omega}$ a $(T-1) \times (T-1)$ unrestricted, positive definite matrix
- Note:
  - Due to a rank deficiency (from first differencing) under the adjusted assumption (see Wooldridge (2002) p.276f) one time period needs to be dropped from the observations
  - The assumption made on $\mathbf{\Lambda}$ carries through to $\mathbf{\Omega}$

1.3) Unobserved Effects Panel Data Models
⇒ Fixed effects (FE) Methods (ctd.)

Fixed effects generalized least squares is carried out in the following steps:

- Estimate β_{FE}
- Drop the last time period for each i and compute the N residual vectors $\hat{\ddot{\mathbf{u}}}_i = \ddot{\mathbf{y}}_i - \ddot{\mathbf{X}}_i\hat{\beta}$ with $dim(\hat{\ddot{\mathbf{u}}}_i) = (T-1)\times 1$
- A consistent estimator for $\mathbf{\Omega}$ is $\hat{\mathbf{\Omega}} = N^{-1}\sum_{i=1}^{N}\hat{\ddot{\mathbf{u}}}_i\hat{\ddot{\mathbf{u}}}_i'$
- The FEGLS estimator is given by

$$\hat{\beta}_{FEGLS} = (\sum_{i=1}^{N}\ddot{\mathbf{X}}_i'\hat{\mathbf{\Omega}}^{-1}\ddot{\mathbf{X}}_i)^{-1}(\sum_{i=1}^{N}\ddot{\mathbf{X}}_i'\hat{\mathbf{\Omega}}^{-1}\ddot{\mathbf{y}}_i) \tag{18}$$

- The asymptotic variance is then given by

$$Avar(\hat{\beta}_{FEGLS}) = (\sum_{i=1}^{N}\ddot{\mathbf{X}}_i'\hat{\mathbf{\Omega}}^{-1}\ddot{\mathbf{X}}_i)$$

1.3) Unobserved Effects Panel Data Models
⇒ HANDS-ON

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1.3) Unobserved Effects Panel Data Models
⇒ First Differencing (FD) Methods

$$y_{it} = \mathbf{x}_{it}\beta + c_i + u_{it} \tag{19}$$

- Instead of using the FE transformation one can also estimate β in Equation (19) by FD methods
- Model and interpretation of β are exactly as in the previous sub-section

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ First Differencing (FD) Methods (ctd.)

To allow estimation within an FD framework, the following assumptions need to hold:

- $E(u_{it}|\mathbf{x}_i, c_i) = 0;\ \ t = 1, 2, ..., T$; with $\mathbf{x}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ...\mathbf{x}_{iT})$
  'Strict exogeneity of the explanatory variables and the unobserved effect'
- $rank(\sum_{t=2}^{T} E(\Delta\mathbf{x}_{it}'\Delta\mathbf{x}_{it})) = K$
  'Regressors are not perfectly linear dependent'
  with $\Delta\mathbf{x}_{it} = \mathbf{x}_{it} - \mathbf{x}_{i,t-1}$
- $E(\mathbf{e}_i\mathbf{e}_i'|\mathbf{x}_{i,1}, ..., \mathbf{x}_{i,T}, c_i) = \sigma_e^2$
  'The first differenced errors are serially uncorrelated and homoscedastic'
  with $e_{it} = \Delta u_{it};\ \ t = 2, ..., T$

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ First Differencing (FD) Methods (ctd.)

- Assuming e_{it} to be serially uncorrelated implies that u_{it} follows a random walk: $u_{it} = u_{i,t-1} + e_{it}$
- A random walk has strong serial dependence. This assumption is thus opposite to the assumption on the error terms under FE methods
- Under the previous assumptions β_{FD} can be estimated consistently and efficiently by the pooled OLS estimator from the regression

$$\Delta y_{it} \text{ on } \Delta \mathbf{x}_{it};\ t = 2, ..., T;\ i = 1, 2, ..., N \tag{20}$$

with $\Delta y_{it} = y_{it} - y_{i,t-1}$

- The asymptotic variance of the FD estimator is given by

$$Avar(\hat{\beta}_{FD}) = \sigma_e^2 (\Delta \mathbf{X}' \Delta \mathbf{X})^{-1}$$

with $\sigma_e^2 = (N(T-1) - K)^{-1} \sum_{i=1}^{N} \sum_{t=2}^{T} \hat{e}_{it}^2$; $\hat{e}_{it} = \Delta y_{it} - \Delta \mathbf{x}_{it} \hat{\beta}_{FD}$

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ First Differencing (FD) Methods (ctd.)

- Under the previous assumption the errors $e_{it} = \Delta u_{it}$ should be serially uncorrelated
- To test whether this assumption holds, the pooled OLS residuals from the regression in Equation (20) can be employed for the following regression:

$$\hat{e}_{it} = \rho \hat{e}_{i,t-1} + \epsilon_{it}; \;\; t = 3, 4, ..., T; \;\; i = 1, 2, ..., N$$

- The null hypothesis $H_0: \;\rho = 0$ can be tested with the usual t-statistic

- If the assumption of serially uncorrelated error terms is violated, the following robust asymptotic variance estimator can be applied:

$$Avar(\hat{\beta}_{FD}) = (\Delta \mathbf{X}'\Delta \mathbf{X})^{-1}(\sum_{i=1}^{N} \Delta \mathbf{X}_i'\hat{\mathbf{e}}_i\hat{\mathbf{e}}_i'\Delta \mathbf{X}_i)(\Delta \mathbf{X}'\Delta \mathbf{X})^{-1}$$

  with $\Delta\mathbf{X}$ the $N(T-1)\times K$ matrix of stacked first differences of $\mathbf{x}_{it}$
- When $T = 2$, FE and FD are identical
- FD easier to implement
- When $T > 2$ the choice between FE and FD hinges on the assumption about the idiosyncratic error terms

1.3) Unobserved Effects Panel Data Models
$\Rightarrow$ Hausman Test

- Since FE is consistent when c_i and $\mathbf{x}_{it}$ are correlated, but RE is inconsistent, a statistically significant difference is interpreted as evidence against the RE model
- To test the null hypothesis H_0 : '$\hat{\beta}_{RE}$ is the correct specification', the following test statistic is asymptotically χ^2_M distributed, with M the first dimension of the vector of parameter estimates

$$H = (\hat{\beta}_{FE} - \hat{\beta}_{RE})'[Avar(\hat{\beta}_{FE}) - Avar(\hat{\beta}_{RE})]^{-1}(\hat{\beta}_{FE} - \hat{\beta}_{RE})$$

- Note that $\hat{\beta}_{RE}$ does not contain the coefficients on time-constant variables or aggregate time variables

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ HANDS-ON

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1.3) Unobserved Effects Panel Data Models
⇒ Additional Remarks

- To implement the notion of 'conditional convergence' in a more sophisticated empirical way, Barro (1991) and Sala-I-Martin (1997) model c_i as a function of, among other things, the schooling rate, fertility rate, investment, government consumption, inflation, openness, and democracy.
- In regression form this can be written as

$$\tilde{y}_{it} = \mathbf{x}_{it}\beta + \tau logy_{i,t-1} + \epsilon_{it}$$

  where $\mathbf{x}$ contains the variables mentioned above
- In a latter part of the lecture we will try to explain countries' output growth with some of these variables
- This kind of equations has been used to estimate the 'determinants of growth'

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ Additional Remarks (ctd.)

- With all models seen so far care needs to be taken to distinguish correlation from causation
  Example: schooling and fertility rate
- Besides the simplicity of regression framework and attractiveness as a bridge between theory and data, several problematic features thus need to be taken into consideration:
  - Endogeneity of variables
  - Measurement error of data
  - In the Solow model, investment is key; if controlled for, other variables such as schooling should have no explanatory power
  - The growth regression framework is derived from a closed-economy Solow model. In reality, economies trade and are no islands.

1.3) Unobserved Effects Panel Data Models $\Rightarrow$ Additional Remarks (ctd.)

- Furthermore, noting that $\tilde{y}_{it} \approx\ log\ y_{it} - log\ y_{i,t-1}$, growth regressions may be written more naturally in an autoregressive framework as

$$log\ y_{it} = c_i + (1+\tau)log\ y_{i,t-1} + \epsilon_{it}$$

- We will investigate this equation at a later point in the lecture
- Including the **x** variables in the equation above can uncover the correlations between these variables and countries' level of output

SUMMARY

- The pooled panel model

$$y_{it} = \alpha + \mathbf{x}_{it}\beta + u_{it}$$

- The unobserved effects panel model

$$y_{it} = \alpha_i + \mathbf{x}_{it}\beta + u_{it}$$

    - Fixed effects?
    - Random effects?

SUMMARY (ctd.)

- The Solow growth model and Barro-type growth regressions

$$\tilde{y}_{it} = \alpha_i(\mathbf{X}_{it}\beta) + \tau y_{i,t-1} + \epsilon_{it}$$

- Pitfalls:
  - endogeneity
  - poor data quality
  - parameter heterogeneity

Digression: The Growth of Growth Theory

DISCUSSION